●●●○○ Level 3 · Mathematics M3.5

Angles, triangles and symmetry

Work out missing angles, draw accurate triangles, and find symmetry in shapes.

Find missing angles on a line, round a point and between parallel lines. Show that a triangle has 180° and a quadrilateral 360°, and find the angles of any polygon. Draw triangles with ruler and compasses, cut an angle in half, find line symmetry and turning symmetry, and draw solids, nets and views.

YOU ARE READY FOR THIS BOOK IF

You can measure an angle with a protractor to the nearest degree.

One step up	M4.3 Pythagoras and trigonometry
This book	**M3.5 Angles, triangles and symmetry**
One step down	M2.5 Shapes and angles

Contents

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M3.5.6 Line symmetry and reflection

M3.5.7 Turning symmetry

M3.5.8 Solids, nets and views

Key words, A to Z

Written and drawn by attnom. Free to read, print and share.

M3.5.1 CHAPTER 1

Angles on a line and round a point

> **Two sticks cross on the ground. One angle is 40°. Can I find the other three without measuring?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** No. I must measure every angle with a protractor.
- ☐ **B** Yes. The opposite angle is 40° and the other two are 140°.
- ☐ **C** Yes. All four angles are 40°.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Shapes and angles, M2.5.4. A right angle is 90° and a full turn is 360°. You also need to measure an angle with a protractor.

BY THE END YOU CAN

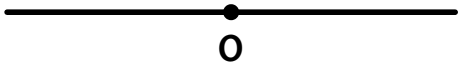
- Find a missing angle on a straight line
- Find a missing angle round a point
- Find the opposite angles where two straight lines cross

Stand still and turn all the way round. That is a full turn. A full turn is 360°. Now turn only half way. That is a half turn, and it is 180°.

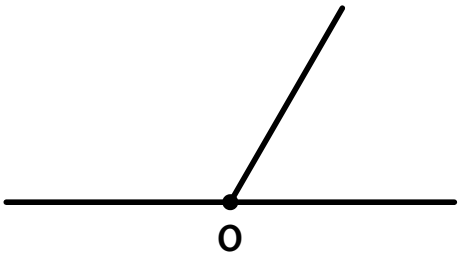
Two lines can meet at one point. We call that point the **vertex**. Angles that share a vertex can fill a straight line or a full turn.

DRAW IT FIRST

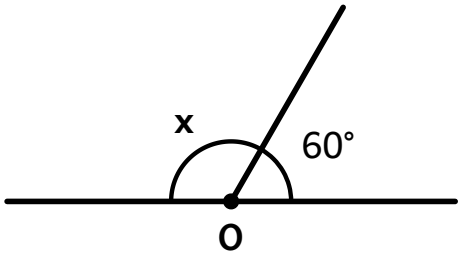
Angles on *a* straight line



1 Draw *a* straight line. Mark *a* point O on it.



2 Draw *a* second line from O. It makes two *a*ngles.



60° + 120° = 180°

3 The two *a*ngles fill the straight line. They *a*dd to 180°.

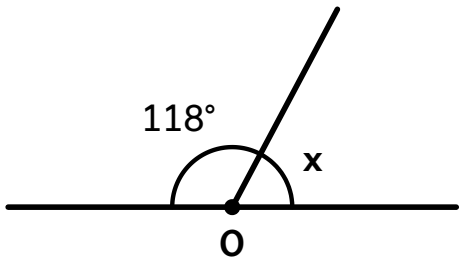
This is the first rule. **<u>Angles on *a* line</u>** *a*dd to 180°. If you know one, take it *a*way from 180° to find the other.

WORKED EXAMPLE

Two angles fill a straight line. One angle is 118°. What is the other angle, x?

1. Angles on a straight line add to 180°.
2. So 118° + x = 180°.
3. x = 180° − 118° = 62°.

Answer: 62°



The angle x is 62°. A protractor on this page would read 62°.

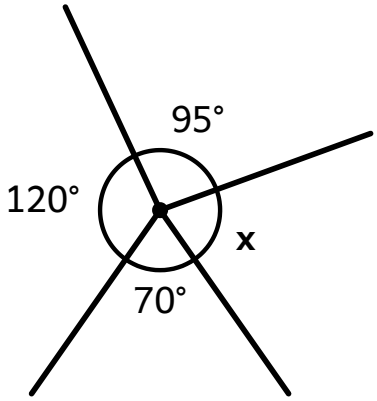
Now go all the way round the vertex. **Angles round a point** fill a full turn, so they add to 360°.

WORKED EXAMPLE

Four angles meet at a point. Three of them are 95°, 120° and 70°. What is the fourth angle, x?

1. Angles round a point add to 360°.
2. The three known angles add to 95° + 120° + 70° = 285°.
3. x = 360° − 285° = 75°.

Answer: 75°



The four angles fill a full turn: 95° + 120° + 70° + 75° = 360°.

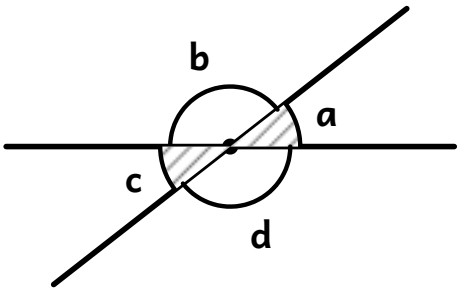
Two straight lines can cross. They make four angles. Angles that face each other across the vertex are **vertically opposite angles**, or opposite angles for short. They are equal.

WORKED EXAMPLE

Two straight lines cross. One angle is 38°. What are the four angles?

1. Call the angles a, b, c and d. Angle a is 38°.
2. Angle b and angle a fill a straight line, so b = 180° − 38° = 142°.
3. Angle c is opposite a, so c = 38°.
4. Angle d is opposite b, so d = 142°.
5. The four angles are 38°, 142°, 38° and 142°.

Answer: 38°, 142°, 38° and 142°



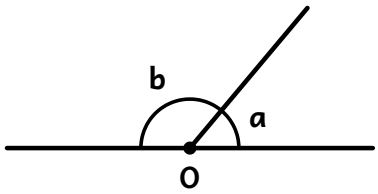
The shaded angles a and c are opposite and equal. Angles b and d are opposite and equal.

SEE IT THREE WAYS

With things

Lay a long stick on the ground. Lean a short stick on it. The short stick makes two angles with the long stick, one on each side.

As a picture



In symbols

$a + b = 180°$. When a is 50°, b is 180° − 50° = 130°.

TRY IT

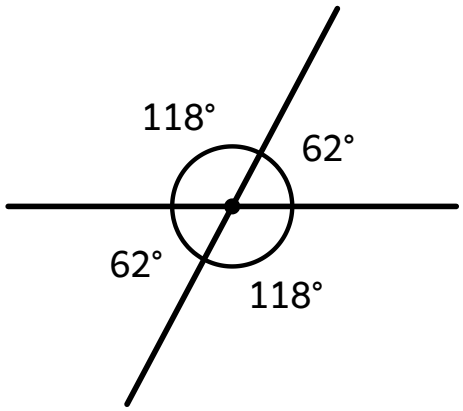
Cross two lines and measure

You need: a sheet of paper, a ruler, a protractor, a pencil.

1. Draw two straight lines that cross.
2. Name the four angles a, b, c and d, going round the vertex.
3. Measure each angle with the protractor.
4. Add a and b. Then add b and c.

What you will see: Opposite angles are the same size. Angles side by side add to 180°. A degree or two of difference is normal when you measure.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~Any two angles that touch add up to 180°.~~

Why it feels right: On a straight line, angles that touch do add up to 180°.

The test that breaks it: Look at the corner of a page. It is 90°. Draw a line from the corner into the page, for example at 30°. You get two angles, 30° and 60°. They touch, but they add to 90°.

What is true: Angles that touch add to 180° only when together they fill a straight line.

KEY WORDS

vertex
the point where two lines meet.

angles on a line
angles that fill a straight line. They add to 180°.

angles round a point
angles that fill a full turn. They add to 360°.

vertically opposite angles
angles that face each other where two straight lines cross. They are equal. We also call them opposite angles.

BACK TO YOUR GUESS

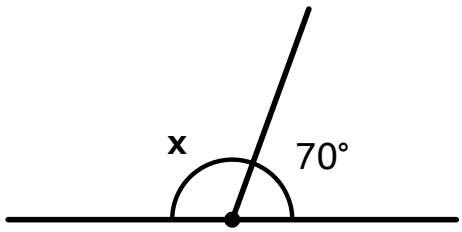
The answer is B: Yes. The opposite angle is 40° and the other two are 140°.

Angles next to each other on a straight line add to 180°. So 180° − 40° = 140°. The angle opposite 40° is also 40°. The four angles are 40°, 140°, 40° and 140°.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Two angles fill a straight line. One is 70°. What is the angle x?



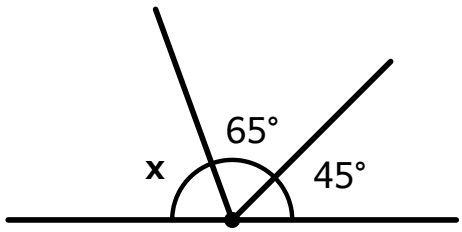
A 110°

B 290°

C 20°

Answer at the back of the book.

2. Three angles fill a straight line. Two of them are 45° and 65°. What is the angle x?



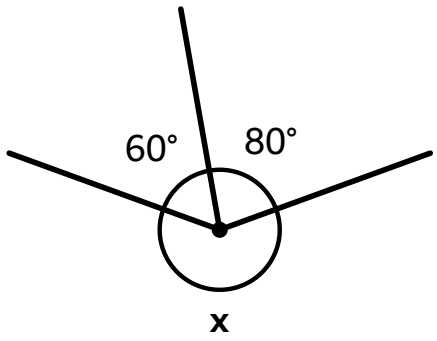
A 110°

B 135°

C 70°

Answer at the back of the book.

3. Three angles fill a full turn round a point. Two of them are 80° and 60°. What is the angle x?



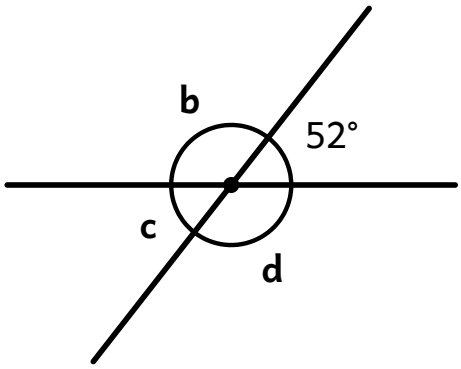
A 140°

B 220°

C 40°

Answer at the back of the book.

4. Two straight lines cross. One angle is 52°. What is the angle c, opposite it?



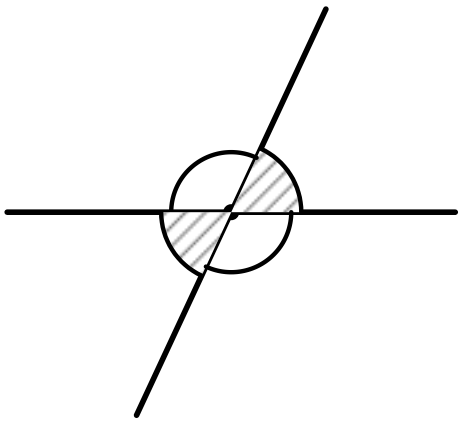
A 52°

B 128°

C 38°

Answer at the back of the book.

5. Which pair of angles is always equal?



A Two angles side by side on a straight line

B Two angles round a point

C Opposite angles where two straight lines cross

Answer at the back of the book.

6. Two angles on a straight line are 117° and x. What is x?

Answer at the back of the book.

7. Three angles round a point are 85°, 140° and 60°. A fourth angle x completes the turn. What is x?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Angles on a straight line add to 180°. Angles round a point add to 360°. Where two straight lines cross, opposite angles are equal.

Draw: Two crossing lines. Mark the four angles and write 40°, 140°, 40° and 140° on them.

M3.5.2 CHAPTER 2

Angles and parallel lines

> **A road crosses two parallel railway tracks. I know one angle. Can I find the other seven?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** No. Every angle must be measured.
- ☐ **B** Yes. Each angle is the one I know, or 180° minus it.
- ☐ **C** Yes. All eight angles are equal.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Shapes and angles, M2.5.1. Parallel lines stay the same distance apart and never meet.

BY THE END YOU CAN

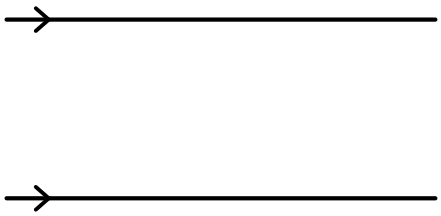
- Find equal angles where a line crosses two parallel lines
- Name alternate and corresponding angles

Parallel lines run side by side. They stay the same distance apart and never meet. We mark them with arrows that point the same way.

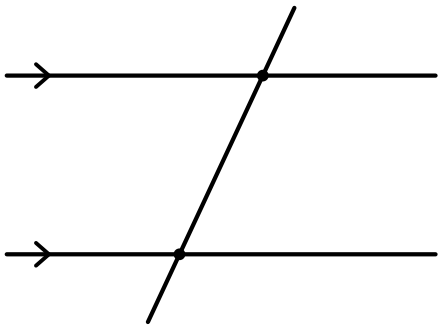
A line that crosses two parallel lines is a **transversal**. It leans the same way at both crossings. So the angles at the top crossing match the angles at the bottom crossing.

DRAW IT FIRST

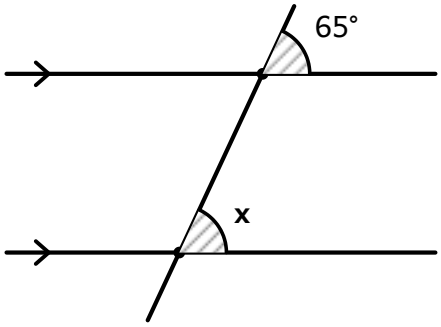
The same place at each crossing



1 Draw two parallel lines. Mark each with an arrow.



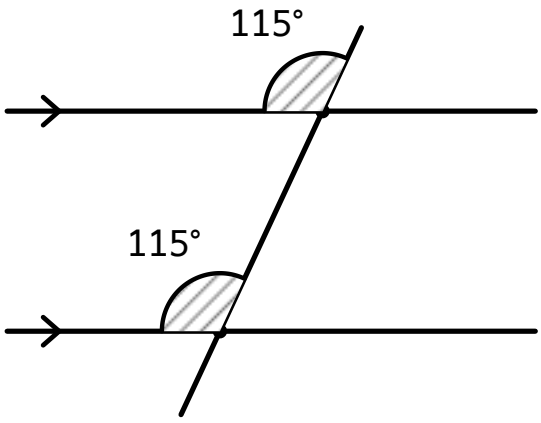
2 Draw a line across both of them.



x = 65°

3 The angle in the same place at each crossing is the same size.

The two shaded angles are in the same place at each crossing. They are **corresponding angles**. Look for the letter F. Corresponding angles between parallel lines are equal.



These corresponding angles make an F shape. Both are 115°.

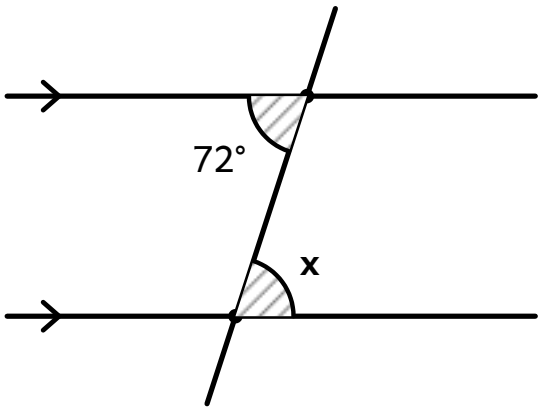
Some angles sit between the parallel lines, on opposite sides of the transversal. They are **alternate angles**. Look for the letter Z. Alternate angles between parallel lines are equal.

WORKED EXAMPLE

The two lines are parallel. One angle is 72°. What is the alternate angle x?

1. Both angles are between the parallel lines.
2. They are on opposite sides of the transversal.
3. They make a Z shape, so they are alternate angles.
4. Alternate angles are equal, so x = 72°.

Answer: 72°



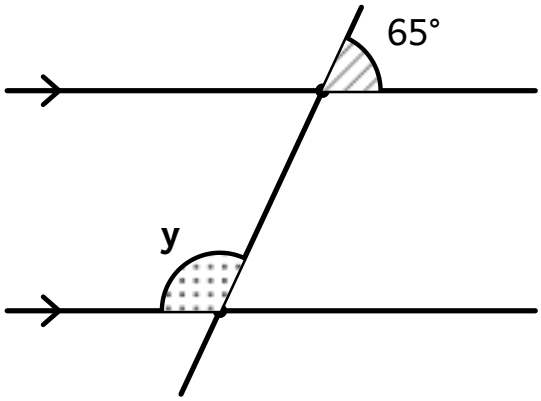
The angle x is 72°. The two angles lie in a Z shape.

WORKED EXAMPLE

The two lines are parallel. The angle at the top is 65°. What is the angle y at the bottom?

1. The angle next to 65° on the top line is 180° − 65° = 115°.
2. The angle y is in the same place at the bottom crossing.
3. So y is a corresponding angle, and y = 115°.

Answer: 115°



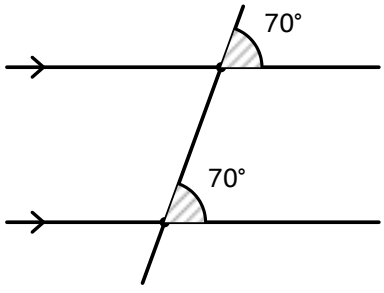
The angle y is 115°. It sits next to the angle that corresponds to 65°, so y = 180° − 65°.

SEE IT THREE WAYS

With things

Draw a line along each long edge of a ruler. The two lines are parallel. Draw a third line across both.

As a picture



In symbols

The angle at the top = the angle in the same place at the bottom. 70° = 70°.

TRY IT

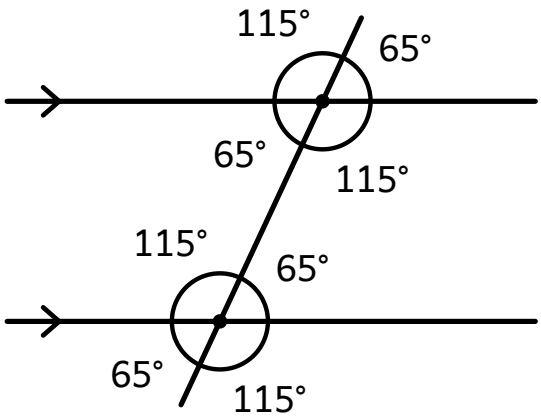
Measure the angles between parallel lines

You need: a sheet of paper, a ruler, a protractor, a pencil.

1. Lay the ruler on the paper. Draw a line along each long edge.
2. Draw a straight line across both lines.
3. Measure the four angles at the top crossing. Then measure the four at the bottom.
4. Compare the angles that are in the same place at each crossing.

What you will see: Each angle matches the one in the same place at the other crossing. Only two sizes appear. A degree or two of difference is normal.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~All eight angles are equal.~~

Why it feels right: The lines are parallel, so everything looks the same.

The test that breaks it: Measure two angles side by side at one crossing. One is small and one is big. They fill a straight line, so they add to 180°.

What is true: Only two sizes appear. Corresponding angles are equal and alternate angles are equal. Angles side by side add to 180°.

KEY WORDS

parallel lines
straight lines that stay the same distance apart and never meet.

transversal
a straight line that crosses two or more other lines.

corresponding angles
angles in the same place at each crossing, in an F shape. Between parallel lines they are equal.

alternate angles
angles between the lines on opposite sides of the transversal, in a Z shape. Between parallel lines they are equal.

BACK TO YOUR GUESS

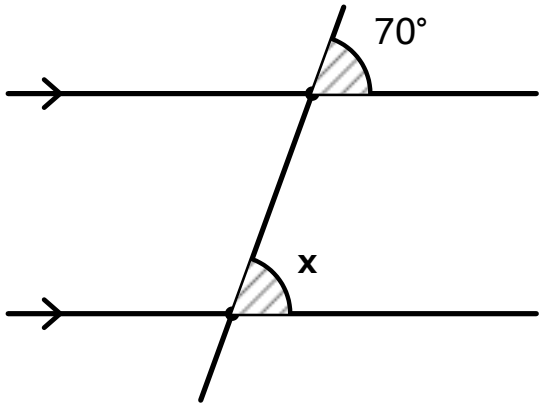
The answer is B: Yes. Each angle is the one I know, or 180° minus it.

The tracks are parallel, so the road meets both of them at the same slant. Only two sizes appear. Four angles are the one you know. The other four are 180° minus it.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. The lines are parallel. The angle x is in the same place as the 70° angle. What is x?



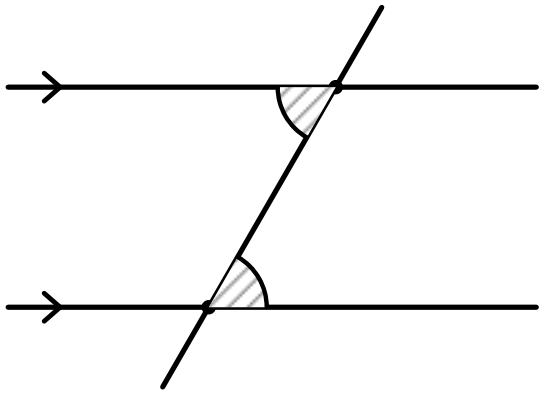
A 70°

B 110°

C 20°

Answer at the back of the book.

2. The lines are parallel. What are the two shaded angles called?



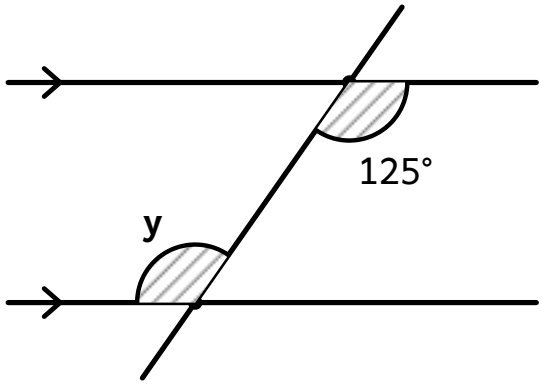
A Corresponding angles

B Opposite angles

C Alternate angles

Answer at the back of the book.

3. The lines are parallel. One angle is 125°. The angle y is its alternate angle. What is y?



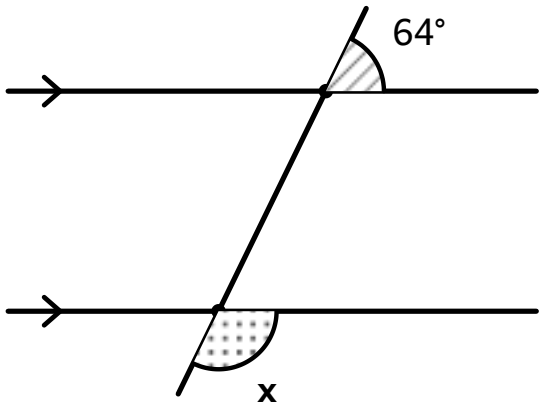
A 55°

B 125°

C 235°

Answer at the back of the book.

4. The lines are parallel. One angle is 64°. What is the angle x?



A 116°

B 64°

C 26°

Answer at the back of the book.

5. Which sentence about two parallel lines and a transversal is true?

   A Alternate angles add to 180°.

   B All eight angles are equal.

   C Corresponding angles are equal.

   Answer at the back of the book.

6. The lines are parallel. An angle at the top is 58°. The angle in the same place at the bottom is x. What is the angle next to x on the straight line?

   Answer at the back of the book.

7. **From an earlier chapter**

   Three angles fill a straight line. Two of them are 38° and 52°. What is the third angle?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A line across parallel lines makes two sizes of angle. Corresponding angles (F) are equal. Alternate angles (Z) are equal. Angles side by side add to 180°.

Draw: Two parallel lines with arrows and a line across them. Shade a pair of corresponding angles and a pair of alternate angles.

M3.5.3 CHAPTER 3

Triangles

Do the three angles of every triangle add up to the same number?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes. They always add up to 180°.
- ☐ **B** No. A big triangle has more angle than a small one.
- ☐ **C** Yes. They always add up to 360°.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Shapes and angles, M2.5.5. Sort triangles by how many of their sides are equal.

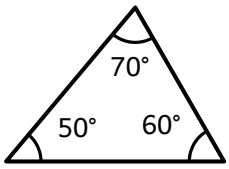
BY THE END YOU CAN

- Show that the angles of a triangle add up to 180°
- Find a missing angle in a triangle
- Name equilateral, isosceles, scalene and right angled triangles

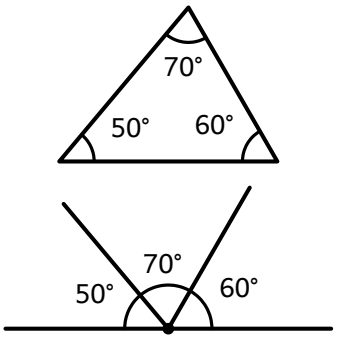
A triangle has three corners and three angles. Add the three angles of any triangle. The answer is always 180°. This is the ***angle sum*** of a triangle.

DRAW IT FIRST

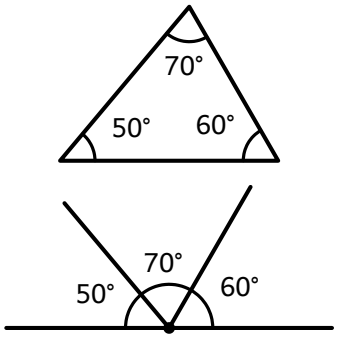
Tear off the corners



1 Draw a triangle. Its angles are 70°, 50° and 60°.



2 Tear off the three corners. Put them side by side with their points together.



60° + 70° + 50° = 180°

3 They fill a straight line. So the three angles add to 180°.

Chapter 2 shows why. Draw a line through the top corner, parallel to the base. The alternate angles match the two bottom corners.

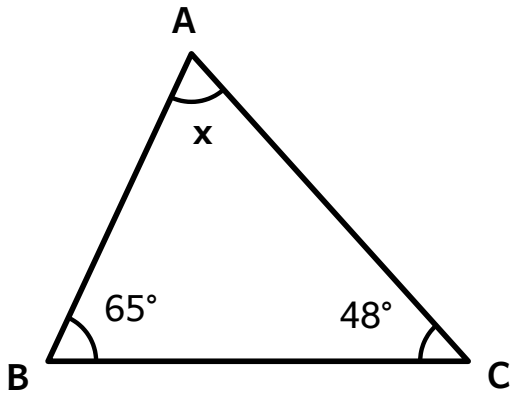
The same works for every triangle. So you can find a missing angle. Add the two angles you know, and take the answer from 180°.

WORKED EXAMPLE

A triangle has angles of 65° and 48°. What is the third angle, x?

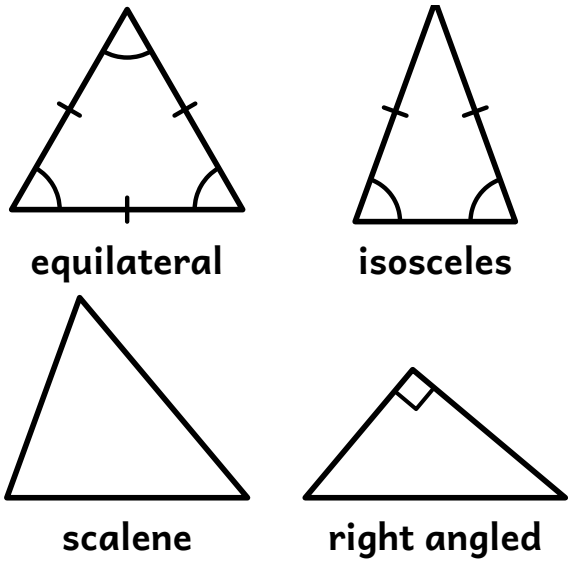
1. The three angles add to 180°.
2. The two known angles add to 65° + 48° = 113°.
3. x = 180° − 113° = 67°.

Answer: 67°



The angle x is 67°: 67° + 65° + 48° = 180°.

Triangles have names. A name tells you how many sides are equal. Small tick marks on sides show which sides are equal.



Ticks show equal sides. Matching arcs show equal angles. The little square shows a right angle.

An **equilateral triangle** has three equal sides and three angles of 60°. An **isosceles triangle** has two equal sides and two equal angles. The equal angles are opposite the equal sides.

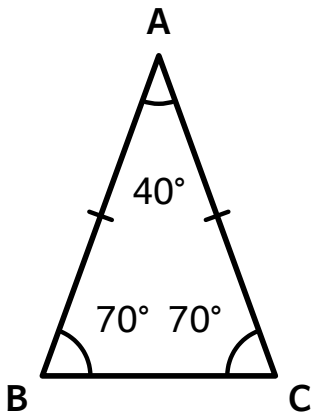
A **scalene triangle** has no equal sides and no equal angles. A **right angled triangle** has one angle of 90°. A right angled triangle can also be isosceles.

WORKED EXAMPLE

An isosceles triangle has an angle of 40° between its two equal sides. What are the other two angles?

1. The three angles add to 180°.
2. The other two angles add to 180° − 40° = 140°.
3. The two equal sides make two equal angles.
4. Each of them is 140° ÷ 2 = 70°.

Answer: 70° and 70°



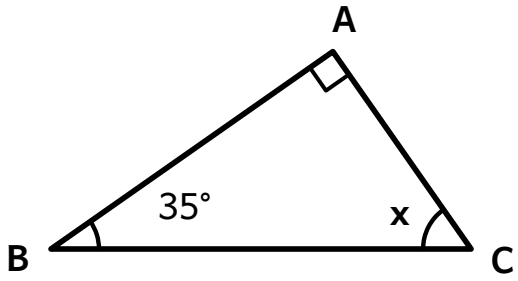
The angles at B and C are equal because the sides AB and AC are equal.

WORKED EXAMPLE

A right angled triangle has an angle of 35°. What is the third angle?

1. One angle is 90°.
2. The three angles add to 180°.
3. The other two angles add to 180° − 90° = 90°.
4. One of them is 35°.
5. The third angle is 90° − 35° = 55°.

Answer: 55°



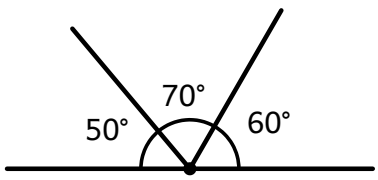
The two angles that are not the right angle add to 90°.

SEE IT THREE WAYS

With things

Draw a triangle on paper. Tear off its three corners. Put them side by side with their points together.

As a picture



In symbols

$a + b + c = 180°$.

For example $70° + 50° + 60° = 180°$.

TRY IT

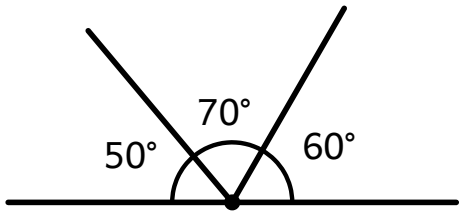
Tear the corners off a triangle

You need: a sheet of paper, a pencil.

1. Draw a triangle on the paper. Make it any shape you like.
2. Mark each corner with a small curved line.
3. Tear off the three corners.
4. Put the corners side by side with their points together.

What you will see: The three corners fill a straight line. That is 180°. It works for every triangle you try.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~A bigger triangle has a bigger angle sum.~~

Why it feels right: A big triangle has longer sides, so it seems to have more of everything.

The test that breaks it: Draw a small triangle and a big triangle. Measure the angles of each and add them up. Both add to about 180°.

What is true: Every triangle has an angle sum of 180°. Longer sides do not make the angles add to more.

KEY WORDS

angle sum
the total of the angles in a shape. For a triangle it is 180°.

equilateral triangle

a triangle with three equal sides and three angles of 60°.

isosceles triangle
a triangle with two equal sides and two equal angles.

scalene triangle
a triangle with no equal sides.

right angled triangle
a triangle with one angle of 90°.

BACK TO YOUR GUESS

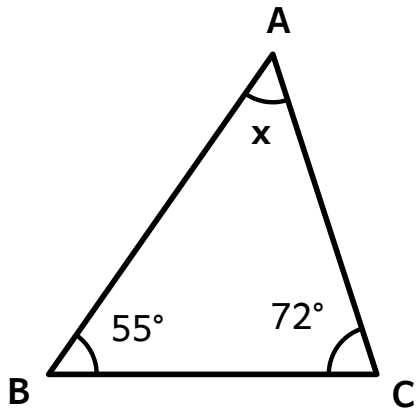
The answer is A: Yes. They always add up to 180°.

Tear off the three corners of a paper triangle and put them side by side. They fill a straight line, which is 180°. This works for every triangle, big or small.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A triangle has angles of 55° and 72°. What is the third angle x?



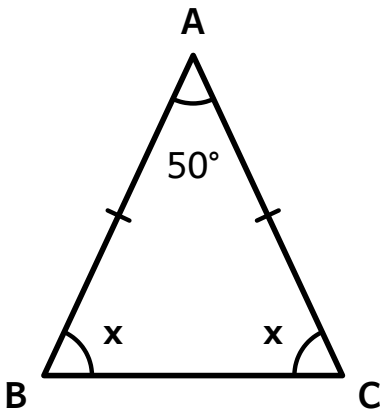
A 127°

B 53°

C 233°

Answer at the back of the book.

2. This isosceles triangle has an angle of 50° between its equal sides. What is the angle x?



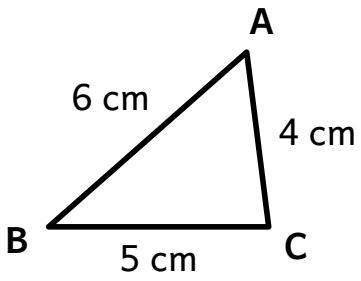
A 130°

B 50°

C 65°

Answer at the back of the book.

3. The sides of this triangle are 4 cm, 5 cm and 6 cm. What kind of triangle is it?



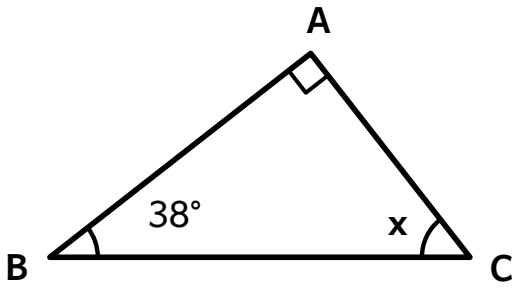
A Scalene triangle

B Isosceles triangle

C Equilateral triangle

Answer at the back of the book.

4. A triangle has a right angle and an angle of 38°. What is the third angle?



A 142°

B 52°

C 90°

Answer at the back of the book.

5. Which three angles can be the angles of a triangle?

A 60°, 60° and 70°

B 100°, 50° and 40°

C 90°, 50° and 40°

Answer at the back of the book.

6. Two angles of a triangle are 48° and 77°. What is the third angle?

Answer at the back of the book.

7. **From an earlier chapter**

Two lines are parallel. An angle at the top is 70°. The angle y is next to the corresponding angle at the bottom, on a straight line. What is y?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

The three angles of a triangle add to 180°. Equilateral: 3 equal sides, all angles 60°. Isosceles: 2 equal sides, 2 equal angles. Scalene: none equal. Right angled: one 90° angle.

Draw: A triangle with its three corners drawn as angles side by side on a straight line. Write 180° under it.

M3.5.4 CHAPTER 4

Angles in polygons

Do the four angles of any four sided shape add up to the same number?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes. They always add up to 360°.
- ☐ **B** Yes. They always add up to 180°.
- ☐ **C** No. Every four sided shape is different.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Shapes and angles, M2.5.2. A polygon is a flat shape with straight sides. A regular polygon has equal sides and equal angles.

BY THE END YOU CAN

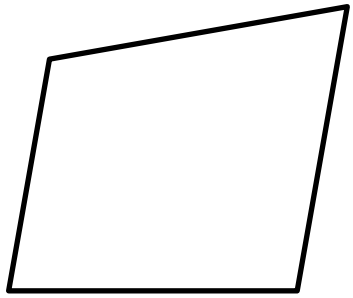
- Show that the angles of a quadrilateral add up to 360°
- Work out the angle sum of any polygon
- Find each angle of a regular hexagon

A **quadrilateral** is a shape with four straight sides. A square, a rectangle and a kite are quadrilaterals. So are many shapes with no name.

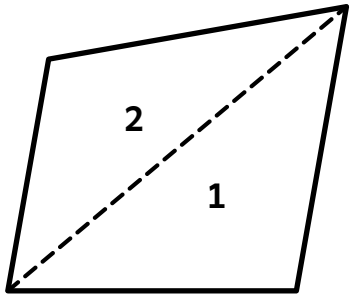
A line that joins two corners that are not next to each other is a **diagonal**. A diagonal cuts a quadrilateral into two triangles.

DRAW IT FIRST

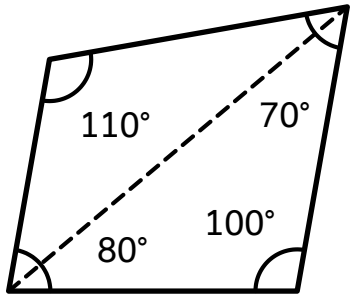
Cut a quadrilateral into two triangles



1 Draw a quadrilateral.



2 Draw one diagonal. It makes two triangles.



2 triangles × 180° = 360°

3 Each triangle has 180°, so the four angles add to 2 × 180° = 360°.

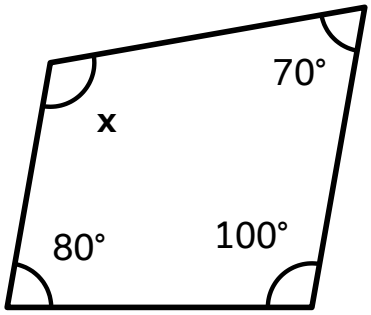
So the angles of every quadrilateral add to 360°. You can use this to find a missing angle.

WORKED EXAMPLE

Three angles of a quadrilateral are 80°, 100° and 70°. What is the fourth angle, x?

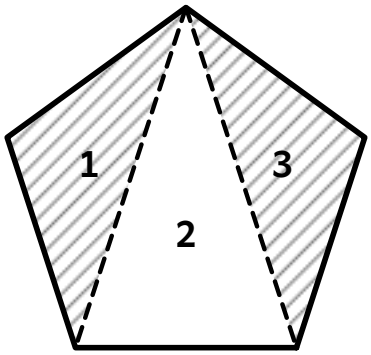
1. The four angles of a quadrilateral add to 360°.
2. The three known angles add to 80° + 100° + 70° = 250°.
3. x = 360° − 250° = 110°.

Answer: 110°

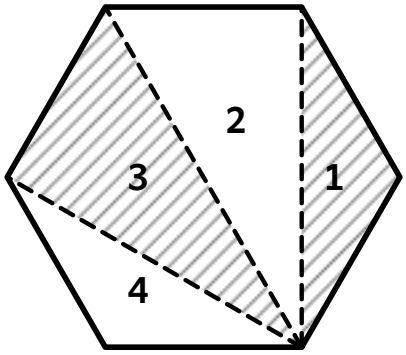


The four angles add to 80° + 100° + 70° + 110° = 360°.

A **polygon** is a flat shape with straight sides. Choose one corner. Draw diagonals from it to every corner it can reach. You get triangles.



A pentagon has 5 sides and makes 3 triangles.



A hexagon has 6 sides and makes 4 triangles.

The number of triangles is always 2 less than the number of sides. So a polygon with n sides makes n − 2 triangles. Its angle sum is (n − 2) × 180°.

sides	triangles	sum
3	1	180°
4	2	360°
5	3	540°
6	4	720°

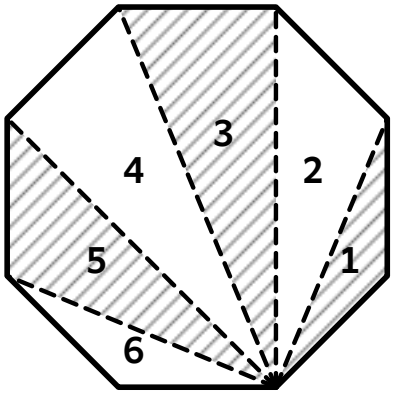
Every extra side adds one more triangle, and 180° more.

WORKED EXAMPLE

What is the angle sum of a polygon with 8 sides?

1. An 8 sided polygon makes 8 − 2 = 6 triangles.
2. Each triangle has an angle sum of 180°.
3. The angle sum is 6 × 180° = 1 080°.

Answer: 1 080°



An octagon makes 6 triangles. 6 × 180° = 1 080°.

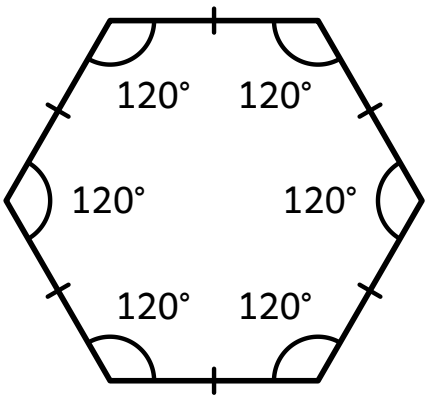
A **regular polygon** has equal sides and equal angles. To find one angle, share the angle sum between all the corners.

WORKED EXAMPLE

What is each angle of a regular hexagon?

1. A hexagon has 6 sides.
2. It makes 6 − 2 = 4 triangles.
3. The angle sum is 4 × 180° = 720°.
4. A regular hexagon has 6 equal angles.
5. Each angle is 720° ÷ 6 = 120°.

Answer: 120°



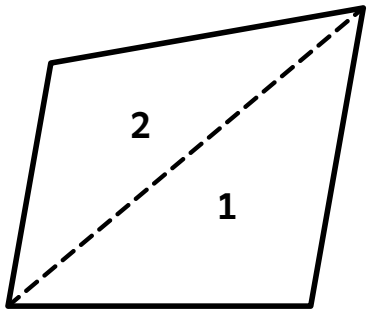
Six angles of 120° make 720°.

SEE IT THREE WAYS

With things

Draw a four sided shape on paper. Draw one diagonal. Cut along it. You have two triangles.

As a picture



In symbols

2 triangles × 180° = 360°. So the four angles add to 360°.

TRY IT

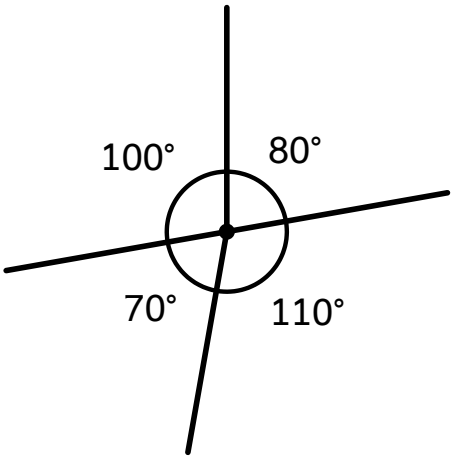
Tear the corners off a quadrilateral

You need: a sheet of paper, a pencil.

1. Draw a four sided shape on the paper. Make it any shape you like.
2. Mark each corner with a small curved line.
3. Tear off the four corners.
4. Put the corners together with their points touching.

What you will see: The four corners fit round a point. That is a full turn, 360°. This works for every four sided shape.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The angles of a quadrilateral add to 180°, like a triangle.~~

Why it feels right: A triangle is the first shape we learn, and its angle sum is 180°.

The test that breaks it: Look at a rectangle. Its four angles are all 90°. Four angles of 90° add to 360°, not 180°.

What is true: A quadrilateral is two triangles joined along a diagonal. So its angle sum is 2 × 180° = 360°.

KEY WORDS

quadrilateral
a shape with four straight sides. Its angles add to 360°.

diagonal
a line that joins two corners that are not next to each other.

polygon
a flat shape with straight sides. With n sides, its angles add to (n − 2) × 180°.

regular polygon
a polygon with all sides equal and all angles equal.

BACK TO YOUR GUESS

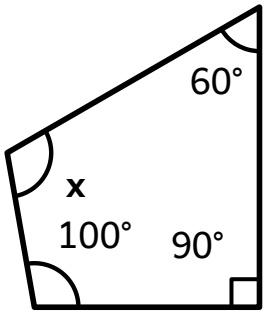
The answer is A: Yes. They always add up to 360°.

Draw a line from one corner to the opposite corner. It cuts the shape into two triangles. Each triangle has 180°, so the four angles add up to 2 × 180° = 360°.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Three angles of a quadrilateral are 100°, 90° and 60°. What is the fourth angle x?



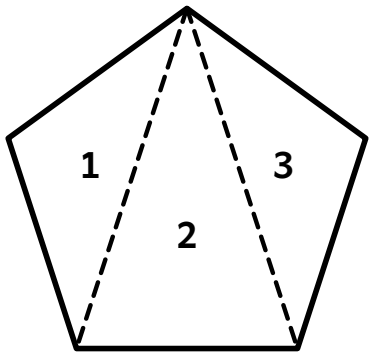
A 250°

B 110°

C 80°

Answer at the back of the book.

2. What is the angle sum of a pentagon?



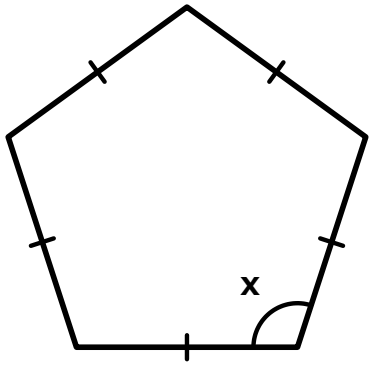
A 900°

B 360°

C 540°

Answer at the back of the book.

3. This polygon is a regular pentagon. What is the angle x?



A 108°

B 540°

C 72°

Answer at the back of the book.

4. Draw all the diagonals from one corner of a polygon with 7 sides. How many triangles do you get?

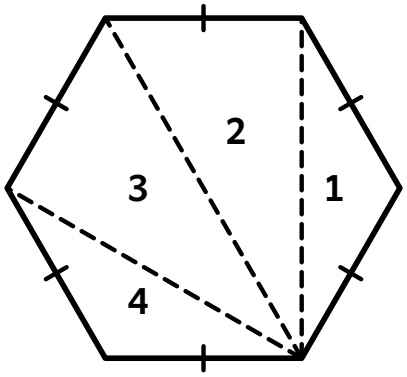
A 7

B 5

C 6

Answer at the back of the book.

5. What is each angle of a regular hexagon?



A 720°

B 60°

C 120°

Answer at the back of the book.

6. What is the angle sum of a polygon with 9 sides?

Answer at the back of the book.

7. **From an earlier chapter**

An isosceles triangle has an angle of 30° between its equal sides. What is each of the other two angles?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A polygon with n sides makes n − 2 triangles. Its angle sum is (n − 2) × 180°. A quadrilateral adds to 360°. Each angle of a regular hexagon is 120°.

Draw: A hexagon cut into four triangles by diagonals from one corner. Write 4 × 180° = 720° beside it.

M3.5.5 CHAPTER 5

Drawing with ruler and compasses

I know the three sides of a triangle. Can I draw it exactly with a ruler and compasses?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Yes. Two arcs show where the third corner goes.
- ☐ **B** No. I need a protractor to measure an angle.
- ☐ **C** Yes. But only if two of the sides are equal.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Shapes and angles, M2.5.4. You need to draw an angle with a protractor and to measure a line with a ruler.

BY THE END YOU CAN

- Draw a triangle from three sides using compasses
- Draw a triangle from two sides and the angle between them
- Cut an angle exactly in half using compasses

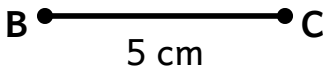
A **construction** is a drawing made with a ruler and **compasses**. Compasses draw circles. A piece of a circle is an ***arc***. You leave every arc on the page, so anyone can see how you drew it.

Be safe. Compasses have a sharp point. Hold them by the top. Keep the point away from your face and from other people. Close them and cover the point when you put them down.

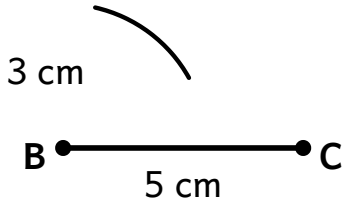
The pictures in this chapter are smaller than real size. Every length in one picture uses the same scale. Draw your own at full size.

DRAW IT FIRST

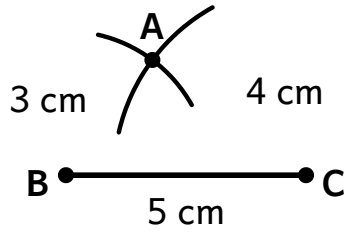
A triangle from three sides



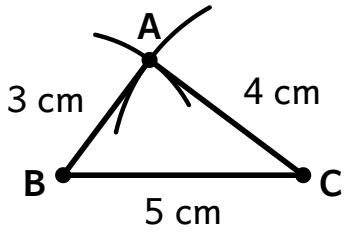
1 Draw the base BC, 5 cm long, with a ruler.



2 Set the compasses to 3 cm. Put the point on B and draw an arc.



3 Set the compasses to 4 cm. Put the point on C and draw an arc that crosses the first. Mark the crossing A.



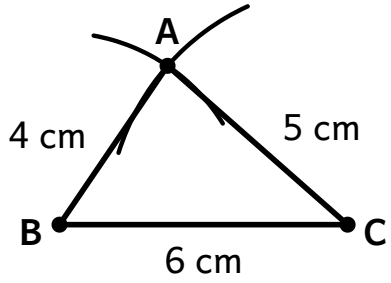
4 Join A to B and A to C with the ruler. Leave the arcs in.

WORKED EXAMPLE

Draw a triangle with sides of 6 cm, 5 cm and 4 cm.

1. Draw the longest side, 6 cm, as the base BC.
2. Set the compasses to 4 cm. Put the point on B and draw an arc.
3. Set the compasses to 5 cm. Put the point on C and draw an arc that crosses the first.
4. Mark the crossing A. Join A to B and A to C.
5. Check with the ruler. AB is 4 cm and AC is 5 cm.

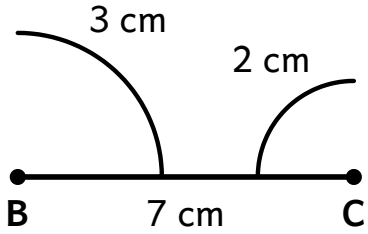
Answer: A triangle with sides of 6 cm, 5 cm and 4 cm



The arcs stay in. They show the compasses were set to 4 cm and 5 cm.

Do the arcs always meet? No. The two shorter sides together must be longer than the longest side. If they are not, the arcs miss each other.

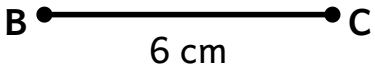
the arcs do not meet



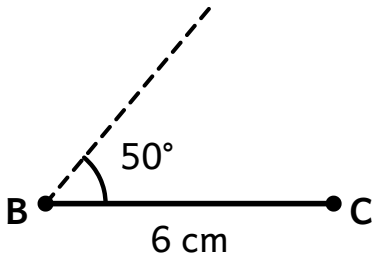
Sides of 7 cm, 3 cm and 2 cm make no triangle. 3 + 2 is less than 7.

DRAW IT FIRST

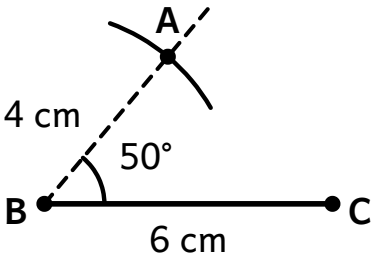
A triangle from two sides and the angle between them



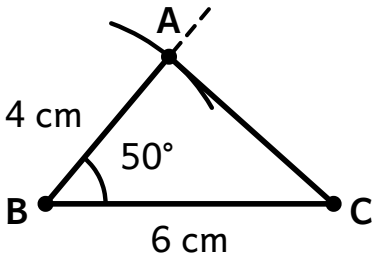
1 Draw the base BC, 6 cm long.



2 Put the protractor on B. Mark 50° from the base and draw a line through the mark.



3 Set the compasses to 4 cm. Put the point on B and draw an arc across the line. Mark the crossing A.



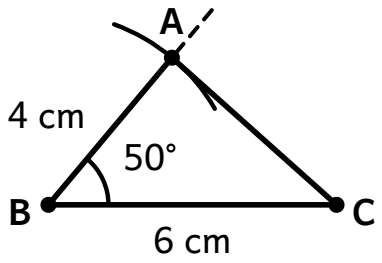
4 Join A to C.

WORKED EXAMPLE

Draw a triangle. Two sides are 6 cm and 4 cm. The angle between them is 50°.

1. Draw the base BC, 6 cm long.
2. At B, use a protractor to mark 50°. Draw a line through the mark.
3. Set the compasses to 4 cm. Put the point on B and draw an arc across the line.
4. Call the crossing A. Join A to C.
5. The angle at B is the angle between the 6 cm side and the 4 cm side.

Answer: A triangle with sides of 6 cm and 4 cm and an angle of 50°

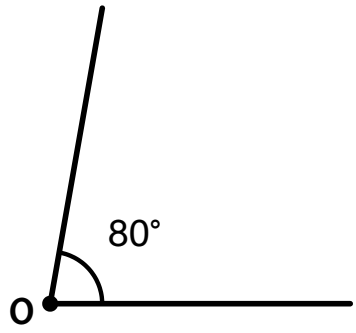


The angle at B is the angle between the two sides you were given.

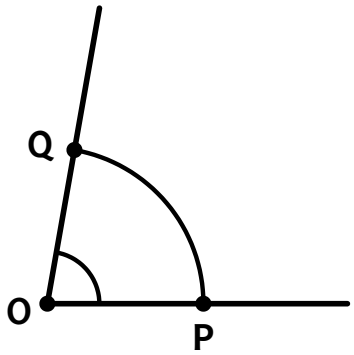
Compasses can also cut an angle in half. We say they **bisect** the angle. The line that does this is the angle bisector.

DRAW IT FIRST

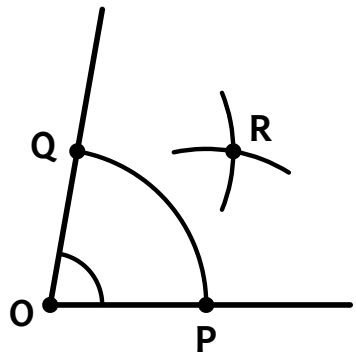
Cut an angle of 80° in half



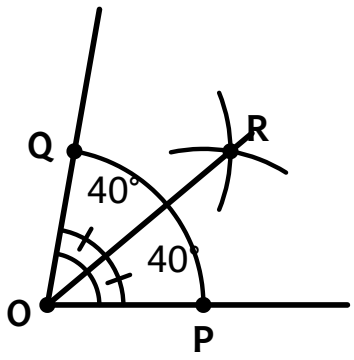
1 Draw an angle of 80°. Its corner is O.



2 Put the compass point on O. Draw an arc that crosses both arms, at P and at Q.



3 Keep the compasses the same. Draw an arc from P and an arc from Q. They cross at R.



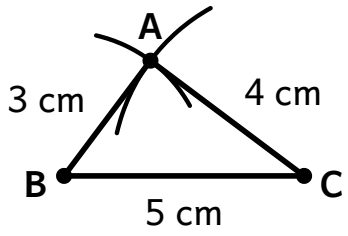
4 Draw a line from O through R. It cuts the angle into two halves of 40°.

SEE IT THREE WAYS

With things

Take two strings, one 3 cm and one 4 cm. Hold their ends on the two ends of a 5 cm line. Pull the loose ends together. They meet at one point.

As a picture



In symbols

3 + 4 is more than 5, so the strings meet. 2 + 3 is less than 7. Strings of 2 cm and 3 cm cannot meet over a 7 cm line.

TRY IT

Draw a triangle with compasses

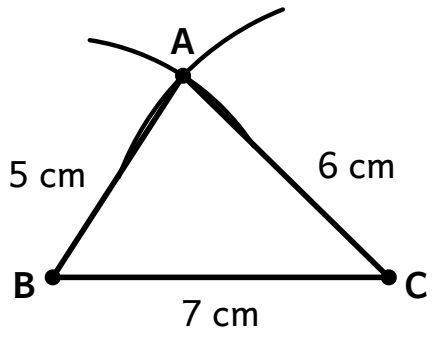
You need: a sheet of paper, a ruler, a pair of compasses, a pencil.

Be safe: Compasses have a sharp point. Hold them by the top. Keep the point away from your face and from other people. Close them and cover the point when you put them down.

1. Draw a line 7 cm long. Call its ends B and C.
2. Set the compasses to 5 cm. Put the point on B and draw an arc above the line.
3. Set the compasses to 6 cm. Put the point on C and draw an arc that crosses the first.
4. Mark the crossing A. Join A to B and A to C. Measure AB and AC with the ruler.

What you will see: AB is 5 cm and AC is 6 cm. Anyone who draws sides of 7 cm, 6 cm and 5 cm gets the same shape.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~You can draw a triangle with any three lengths.~~

Why it feels right: Any three sticks seem as if they should make a triangle.

The test that breaks it: Try sides of 2 cm, 3 cm and 7 cm. Draw the 7 cm base and the two arcs. They never meet.

What is true: The two shorter sides together must be longer than the longest side.

KEY WORDS

construction
a drawing made with a ruler and compasses, with the arcs left in.

compasses
a tool that draws circles and arcs. It has a sharp point.

arc
a piece of a circle.

bisect
to cut exactly in half. The line that bisects an angle is the angle bisector.

BACK TO YOUR GUESS

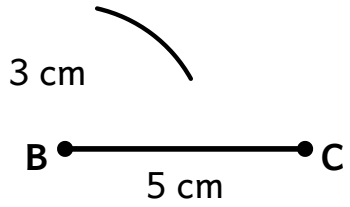
The answer is A: Yes. Two arcs show where the third corner goes.

Draw one side as the base. Set the compasses to the second side and draw an arc from one end. Set them to the third side and draw an arc from the other end. The arcs cross at the third corner.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. You draw a triangle with sides of 5 cm, 3 cm and 4 cm. The base BC is 5 cm. You draw an arc from B with the compasses set to 3 cm. What do you set them to for the arc from C?



A 5 cm

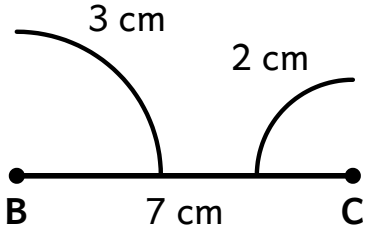
B 3 cm

C 4 cm

Answer at the back of the book.

2. Can you draw a triangle with sides of 7 cm, 3 cm and 2 cm?

the arcs do not meet



A Yes. Any three lengths make a triangle.

B No. The arcs do not meet.

C Yes. I only need a longer ruler.

Answer at the back of the book.

3. Which three lengths can be the sides of a triangle?

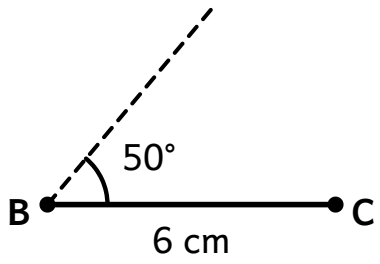
A 5 cm, 6 cm, 10 cm

B 2 cm, 2 cm, 4 cm

C 3 cm, 4 cm, 8 cm

Answer at the back of the book.

4. A triangle has a base BC of 6 cm and a side BA of 4 cm. The angle at B is 50°. How do you find the corner A?



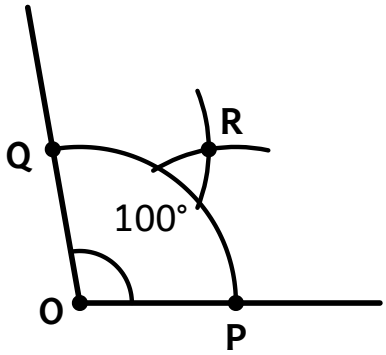
A Draw a line at 50° from B. Set the compasses to 6 cm and draw an arc about B across the line.

B Draw an arc about B with the compasses set to 4 cm. Draw an arc about C that crosses it.

C Draw a line at 50° from B. Set the compasses to 4 cm and draw an arc about B across the line.

Answer at the back of the book.

5. The arcs are drawn from P and from Q and they cross at R. The line from O through R cuts the angle in half. What is each half?



A 200°

B 50°

C 25°

Answer at the back of the book.

6. An angle of 84° is cut in half with compasses. How big is each half?

Answer at the back of the book.

7. **From an earlier chapter**

A right angled triangle has an angle of 28°. What is the third angle?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Three sides: draw the base, then two arcs that cross at the third corner. Two sides and an angle: mark the angle, then use an arc for the length. Arcs from P and Q cut an angle in half.

Draw: A base BC with two arcs crossing at A. Join A to B and A to C. Write the three side lengths in cm.

M3.5.6 CHAPTER 6

Line symmetry and reflection

> **In how many ways can I fold a rectangle so that both halves match?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Two ways.
- ☐ **B** One way only.
- ☐ **C** Four ways: two across the middle and two along the diagonals.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Shapes and angles, M2.5.6. Find a line of symmetry by folding paper. Complete a drawing so both sides match.
- Level 2, Shapes and angles, M2.5.8. Give a position on a grid by column and row.

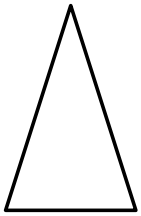
BY THE END YOU CAN

- Find all the lines of symmetry of a shape
- Draw the reflection of a shape in a mirror line

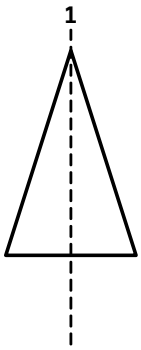
A **line of symmetry** cuts a shape into two halves that match. Fold the shape along the line. The two halves lie exactly on top of each other.

DRAW IT FIRST

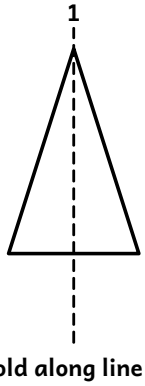
Test for *a* line of symmetry



1 Draw the shape on paper.

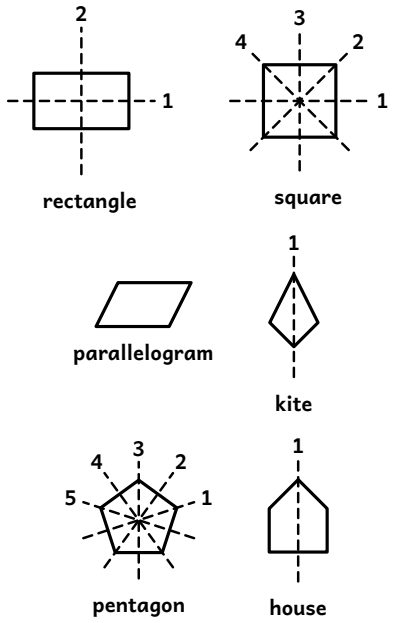


2 Draw *a* line you think is *a* line of symmetry.



3 Fold along the line. If the two halves match exactly, it is *a* line of symmetry.

A shape can have no line of symmetry, one line, or many. We number each line so none is counted twice.



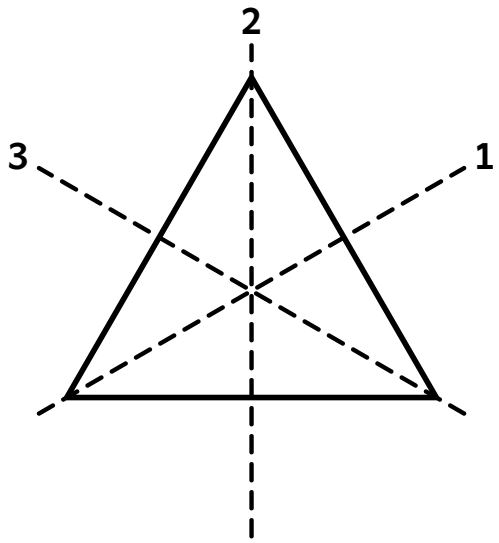
The parallelogram has no line of symmetry. Its halves match only when you turn one over.

WORKED EXAMPLE

How many lines of symmetry does an equilateral triangle have?

1. Each corner has a line from it to the middle of the opposite side.
2. Fold along that line. The two halves match.
3. The triangle has 3 corners, so it has 3 lines.
4. An equilateral triangle has 3 lines of symmetry.

Answer: 3

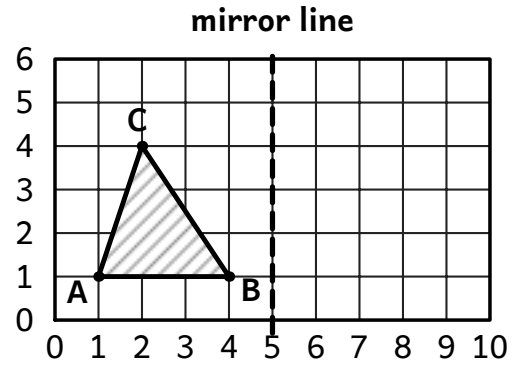


Three lines, one from each corner.

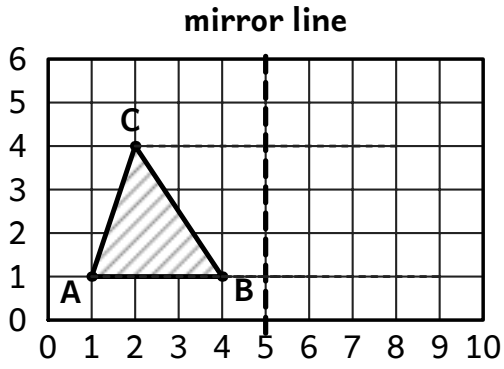
A line of symmetry works like a mirror. The shape on one side is the **reflection** of the shape on the other. The reflection is called the **image**.

DRAW IT FIRST

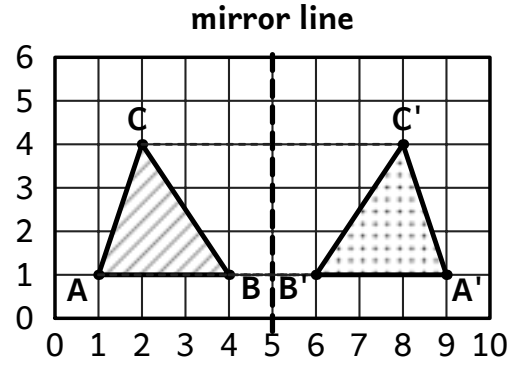
Draw *a* reflection



1 Draw the shape and the mirror line on *a* grid.



2 From each corner, draw *a* line straight *a*cross the mirror line.



3 Mark each new corner the same distance from the mirror line. Join them to make the image.

Each corner of the image is *a*s far from the mirror line *a*s its matching corner. The image is the same size *a*nd shape.

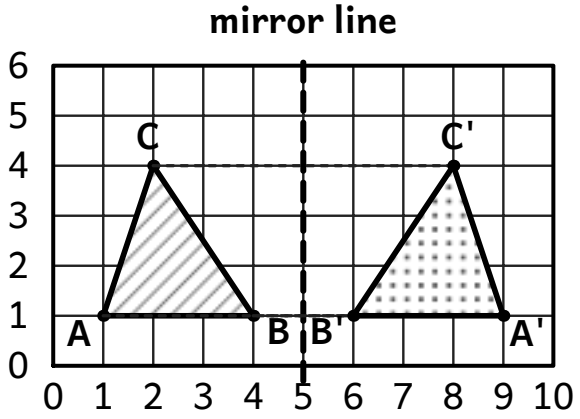
A point such *a*s (2, 3) means 2 *a*cross *a*nd 3 up. The mirror line x = 5 is the upright line through 5 on the bottom scale. The line y = 3 lies flat through 3 on the side scale.

WORKED EXAMPLE

Reflect the triangle with corners A(1, 1), B(4, 1) and C(2, 4) in the mirror line x = 5.

1. A is 5 − 1 = 4 squares left of the mirror line.
2. So A' is 4 squares to the right, at 5 + 4 = 9.
3. B is 1 square left of the line, so B' is at 5 + 1 = 6.
4. C is 3 squares left of the line, so C' is at 5 + 3 = 8.
5. The heights do not change. The image has corners (9, 1), (6, 1) and (8, 4).

Answer: (9, 1), (6, 1) and (8, 4)



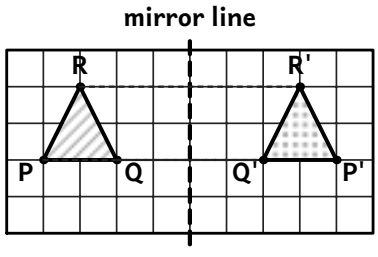
A is 4 squares from the mirror line and A' is 4 squares on the other side.

SEE IT THREE WAYS

With things

Stand a small mirror upright on a line drawn on paper. Put a pencil on one side of the line. Look in the mirror. The pencil seems to stand the same distance behind the mirror.

As a picture



In symbols

A point at 1 is 4 squares from the mirror at 5. Its image is 4 squares the other side, at 5 + 4 = 9.

TRY IT

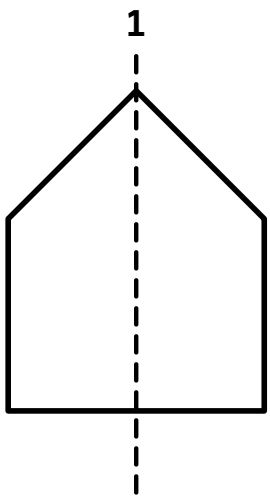
Make a symmetrical shape

You need: a sheet of thin paper, a pencil.

1. Fold the paper in half and open it out. The fold is the mirror line.
2. On one side, draw half of a shape. Let it touch the fold.
3. Fold the paper again. Hold it up to a window.
4. Trace the lines you can see through the paper onto the other side. Open it out.

What you will see: You see the whole shape. The two halves match, and the fold is its line of symmetry.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~A diagonal of a rectangle is a line of symmetry.~~

Why it feels right: It cuts the rectangle into two halves of the same size and shape.

The test that breaks it: Cut out a rectangle of paper. Fold it along a diagonal. The corners do not meet, because the two halves do not lie on top of each other.

What is true: A line of symmetry gives two halves that fold onto each other exactly. Halves of equal size are not enough.

KEY WORDS

line of symmetry
a line that cuts a shape into two halves that fold exactly onto each other.

mirror line
the line a shape is reflected in.

reflection
the shape you see on the other side of a mirror line.

image
the shape that a reflection makes.

BACK TO YOUR GUESS

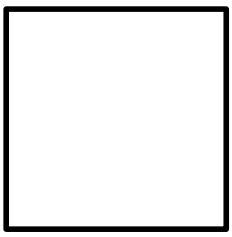
The answer is A: Two ways.

You can fold a rectangle across the middle in two ways: top to bottom and left to right. A fold along a diagonal does not work. The two halves do not lie on top of each other.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. How many lines of symmetry does a square have?



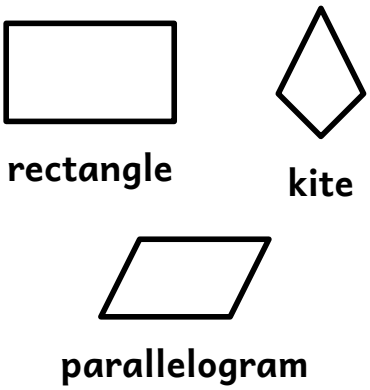
A 2

B 1

C 4

Answer at the back of the book.

2. Which of these shapes has no line of symmetry?



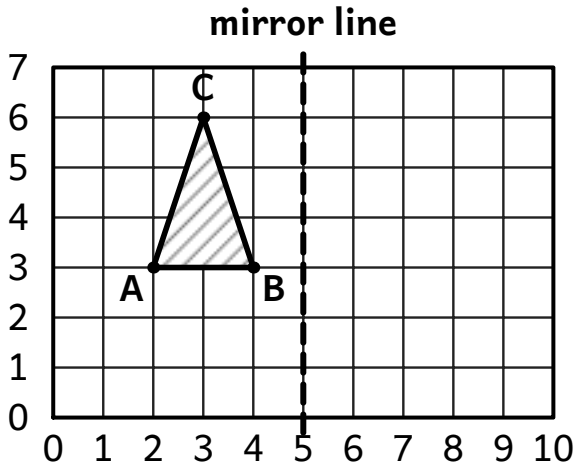
A Rectangle

B Parallelogram

C Kite

Answer at the back of the book.

3. The triangle ABC is reflected in the mirror line x = 5. A is at (2, 3). Where is A'?



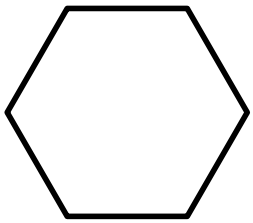
A (8, 3)

B (3, 8)

C (7, 3)

Answer at the back of the book.

4. How many lines of symmetry does a regular hexagon have?



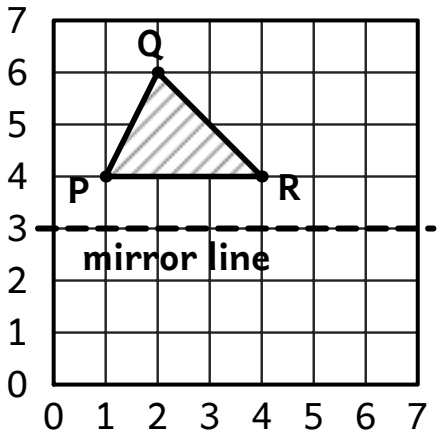
A 3

B 12

C 6

Answer at the back of the book.

5. The triangle is reflected in the mirror line y = 3. The corner P is at (1, 4). Where is P'?



A (1, 5)

B (1, 2)

C (1, 3)

Answer at the back of the book.

6. A regular polygon has 7 sides. How many lines of symmetry does it have?

Answer at the back of the book.

7. **From an earlier chapter**

What is each angle of a regular octagon?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A line of symmetry gives two halves that fold onto each other. A reflection puts every point the same distance from the mirror line on the other side. A regular polygon has as many lines of symmetry as sides.

Draw: A triangle on a grid, a dashed mirror line, and its reflection. Join each corner to its image with a dashed line.

M3.5.7 CHAPTER 7

Turning symmetry

> **A square can be turned and still look the same. How many times does it fit on itself in one full turn?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Four times, every quarter turn.
- ☐ **B** Twice, after a half turn and a full turn.
- ☐ **C** Once, after the full turn.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Shapes and angles, M2.5.6. Find lines of symmetry by folding paper. Turning symmetry is a different kind of symmetry.
- Level 2, Shapes and angles, M2.5.8. Give a position on a grid by column and row.

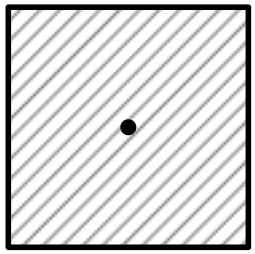
BY THE END YOU CAN

- Find the order of turning symmetry of a shape
- Make a pattern with turning symmetry of order 4

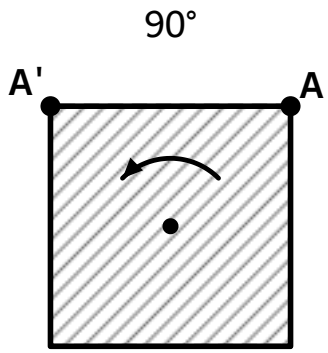
A shape has **turning symmetry** when it fits on itself after a part turn. You turn it about a point in its middle. That point is the **centre of turning**.

DRAW IT FIRST

Turn *a* square *a* quarter turn

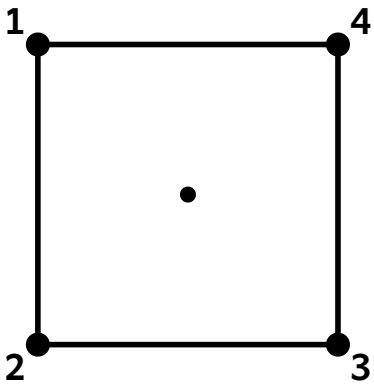


1 Draw the square and mark its centre with *a* dot.



2 Mark one corner A. Turn the square *a* quarter turn, 90°, about the centre. Corner A moves to A'. The turned square fits exactly on the first.

The **order** of turning symmetry is how many times *a* shape fits on itself in one full turn. A square fits 4 times, so its order is 4. The smallest turn that fits is 360° ÷ 4 = 90°.



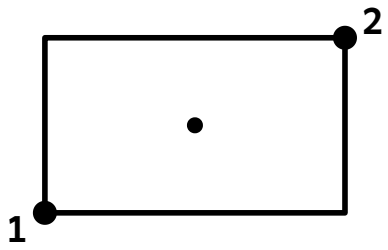
The marked corner lands on *a* corner 4 times, so the order is 4.

WORKED EXAMPLE

What is the order of turning symmetry of a rectangle that is not a square?

1. Mark one corner of the rectangle.
2. Turn the rectangle a half turn, 180°. It fits on itself.
3. A quarter turn does not fit. The long sides would land on the short sides.
4. After the full turn it fits again.
5. It fits 2 times in one full turn, so the order is 2.

Answer: 2



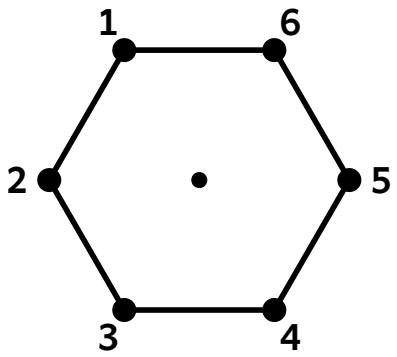
The marked corner lands on a corner 2 times, so the order is 2.

WORKED EXAMPLE

A regular hexagon fits on itself in a full turn. What is its order, and the smallest turn?

1. A regular hexagon has 6 equal sides and 6 equal angles.
2. Each corner can move to the next corner and the shape still fits.
3. So it fits 6 times in one full turn.
4. The order is 6.
5. The smallest turn is 360° ÷ 6 = 60°.

Answer: Order 6 and a smallest turn of 60°



Six turns of 60° make one full turn of 360°.

Some shapes fit only after the full turn. Their order is 1. We say they have no turning symmetry. An isosceles triangle is one. A shape can have turning symmetry and no line of symmetry.

shape	lines	order
square	4	4
rectangle	2	2
parallelogram	0	2
kite	1	1
letter Z	0	2

Lines of symmetry and the order of turning symmetry are different counts.

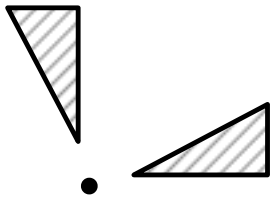
You can use turning symmetry to make a pattern. Draw one shape. Copy it round a centre point, one turn at a time. For order 4, each copy is turned 90° from the last. You draw 4 copies in all.

DRAW IT FIRST

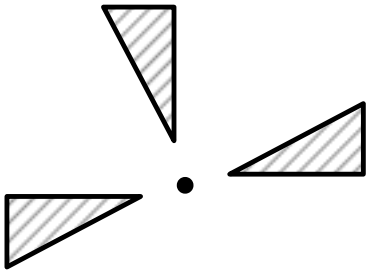
Make a pattern of order 4



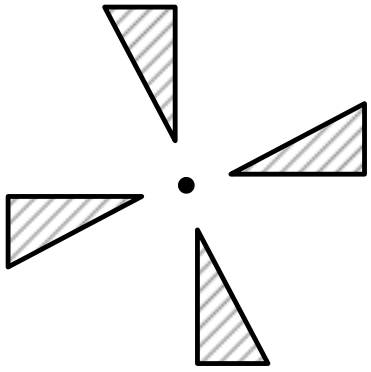
1 Mark a centre point. Draw a shape beside it.



2 Turn the paper 90° about the centre. Draw the shape again.



3 Turn another 90°. Draw the shape a third time.



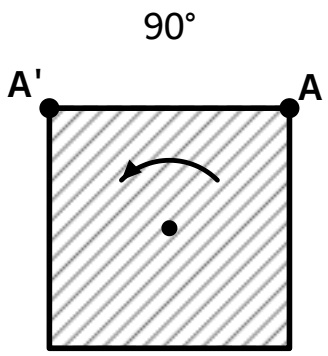
4 Turn another 90°. Draw it a fourth time. The pattern has order 4.

SEE IT THREE WAYS

With things

Trace a square on paper. Hold the tracing on the square with a pencil point at its centre. Turn the tracing a quarter turn.

As a picture



In symbols

Order 4. The smallest turn is 360° ÷ 4 = 90°.

TRY IT

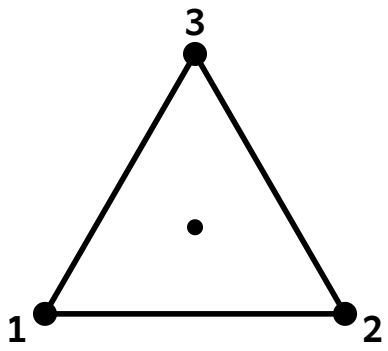
Find the order with tracing paper

You need: a shape drawn on paper, such as a triangle with equal sides, a sheet of thin paper, a pencil.

1. Lay the thin paper on the shape. Trace the shape and mark its centre.
2. Hold the tracing on the shape with a pencil point on the centre.
3. Turn the tracing slowly. Each time it fits on the shape, count 1.
4. Stop when the tracing is back where it started. The count is the order.

What you will see: A triangle with equal sides fits 3 times in a full turn, so its order is 3. A rectangle fits 2 times.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~A parallelogram has no symmetry.~~

Why it feels right: It has no line of symmetry, so it seems to have no symmetry at all.

The test that breaks it: Trace a parallelogram. Hold the tracing at the centre and turn it a half turn. It fits on the shape.

What is true: A parallelogram has turning symmetry of order 2. Line symmetry and turning symmetry are different.

KEY WORDS

turning symmetry
a shape has it when it fits on itself after a part turn.

centre of turning
the point a shape is turned about.

order
how many times a shape fits on itself in one full turn.

BACK TO YOUR GUESS

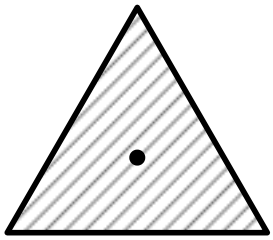
The answer is A: Four times, every quarter turn.

Turn a square about its centre. After a quarter turn it fits on itself. It fits again after a half turn, after three quarters of a turn, and after the full turn. That is 4 times.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is the order of turning symmetry of a triangle with three equal sides?



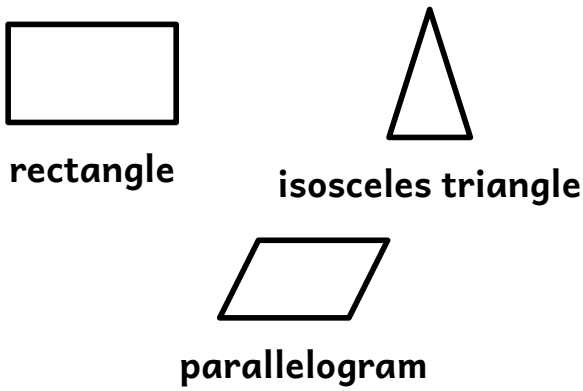
A 1

B 6

C 3

Answer at the back of the book.

2. Which shape has turning symmetry of order 2 but no line of symmetry?



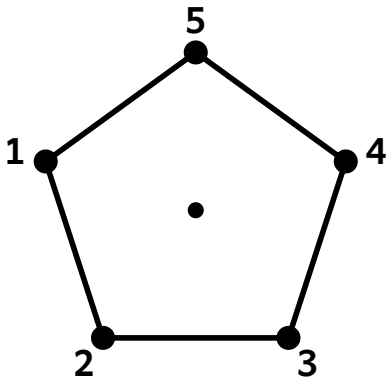
A Rectangle

B Parallelogram

C Isosceles triangle

Answer at the back of the book.

3. A shape fits on itself 5 times in one full turn. What is the smallest turn that fits it?



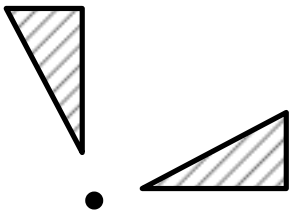
A 72°

B 90°

C 60°

Answer at the back of the book.

4. You copy a shape 4 times round a centre to make a pattern of order 4. How far do you turn it each time?



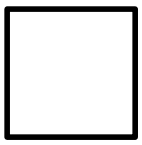
A 45°

B 180°

C 90°

Answer at the back of the book.

5. Which shape has a line of symmetry but no turning symmetry?



square



isosceles triangle



parallelogram

A Square

B Isosceles triangle

C Parallelogram

Answer at the back of the book.

6. A regular polygon has 6 sides. What is its order of turning symmetry?

Answer at the back of the book.

7. **From an earlier chapter**

The point (2, 3) is reflected in the mirror line x = 6. Where is the image?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

The order of turning symmetry is how many times a shape fits on itself in one full turn. The smallest turn is 360° ÷ order. A pattern of order 4 copies a shape every 90°.

Draw: A square with a dot at its centre. Mark one corner 1, 2, 3 and 4 as it turns a quarter turn each time.

M3.5.8 CHAPTER 8

Solids, nets and views

> **A cube has 6 faces, 8 corners and 12 edges. Is there a rule that links the three numbers?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** No. The three numbers have nothing to do with each other.
- ☐ **B** Yes. The three numbers always add up to 26.
- ☐ **C** Yes. Faces plus corners take away edges is 2 for every solid like this.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Shapes and angles, M2.5.7. Count the faces, edges and corners of a cube and a prism. Match a solid to its net.

BY THE END YOU CAN

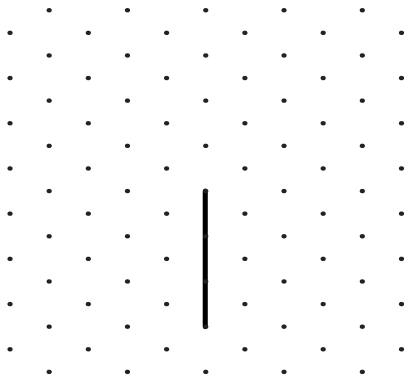
- Draw a cube on dotted paper
- Draw a solid as seen from the front, the side and the top
- Check that faces + corners − edges = 2 for a solid

A solid has flat **faces**. Two faces meet along an **edge**. Edges meet at a **corner**. A cube has 6 faces, 12 edges and 8 corners.

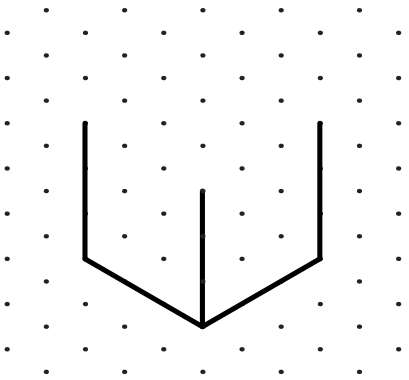
Isometric dotted paper has dots in a pattern of triangles. You draw lines that go straight up, up to the right and up to the left. Each step from dot to dot is 1 cm here.

DRAW IT FIRST

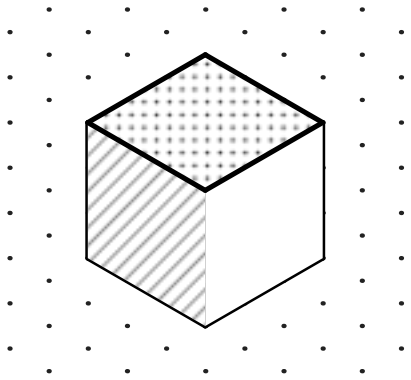
Draw a cube on dotted paper



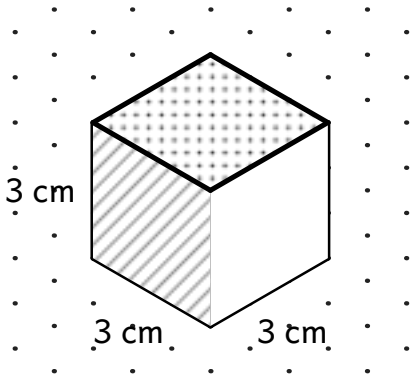
1 Draw one upright line, 3 steps long.



2 From its foot draw a line up to the right and a line up to the left. Each is 3 steps long. Add an upright line at the end of each.

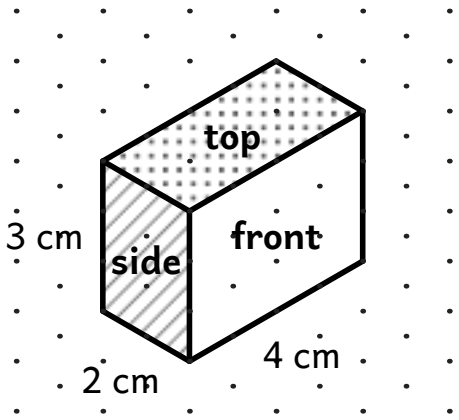


3 Join the tops to make the top face. Use four lines in all, each 3 steps long.



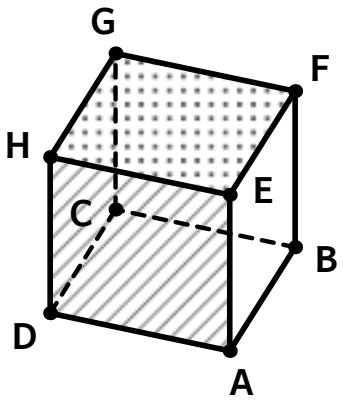
4 Check: 9 lines, each 3 steps long. The corner at the back is hidden right behind the front top corner.

You can draw a cuboid in the same way. Its three lengths can be different. A cuboid is a box.



A cuboid 4 cm long, 2 cm wide and 3 cm high.

Now count the parts of a cube. Letters name the corners. Dashed lines show the edges you cannot see.



A cube has 8 corners, 12 edges (9 solid and 3 dashed) and 6 faces. This cube is turned a little and is not on dotted paper, so the hidden corner shows.

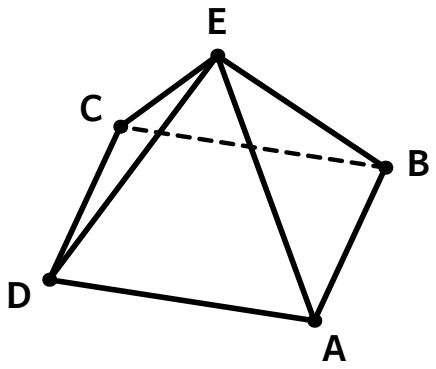
Take a solid with flat faces and no hole through it. Count its **faces**, corners and edges. Then work out faces + corners − edges. The answer is always 2.

WORKED EXAMPLE

Check that faces + corners − edges = 2 for *a* cube.

1. A cube has 6 faces.
2. It has 8 corners.
3. It has 9 edges you can see and 3 at the back, so 12 edges.
4. Faces + corners − edges = 6 + 8 − 12 = 2.

Answer: 2



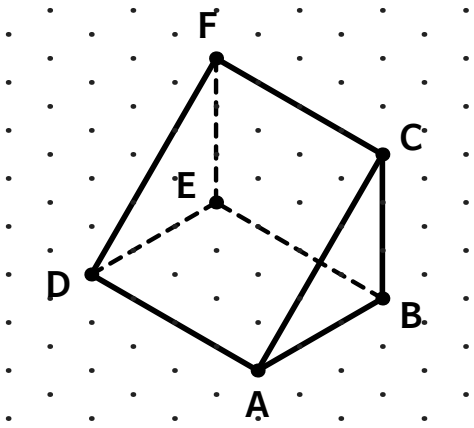
A square based pyramid: 1 square face and 4 triangle faces.

WORKED EXAMPLE

Check that faces + corners − edges = 2 for *a* square based pyramid.

1. It has 1 square face and 4 triangle faces. That is 5 faces.
2. It has 4 corners on the base and 1 at the top. That is 5 corners.
3. It has 4 edges on the base and 4 edges up to the top. That is 8 edges.
4. Faces + corners − edges = 5 + 5 − 8 = 2.

Answer: 2



A triangular prism: 2 triangle faces and 3 rectangle faces.

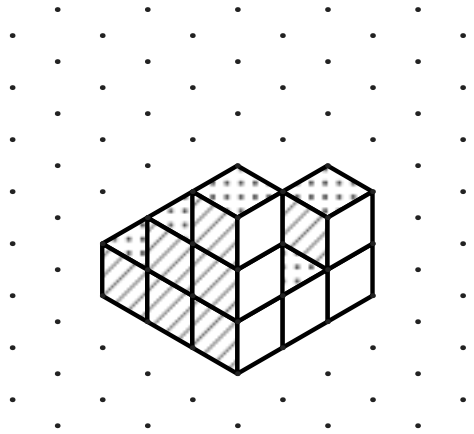
WORKED EXAMPLE

Check that faces + corners − edges = 2 for a triangular prism.

1. It has 2 triangle faces and 3 rectangle faces. That is 5 faces.
2. It has 3 corners at each end. That is 6 corners.
3. It has 3 edges at each end and 3 edges between the ends. That is 9 edges.
4. Faces + corners − edges = 5 + 6 − 9 = 2.

Answer: 2

A solid looks different from the front, from the side and from above. Each of these is a **view**. You draw a view as a flat shape made of squares.



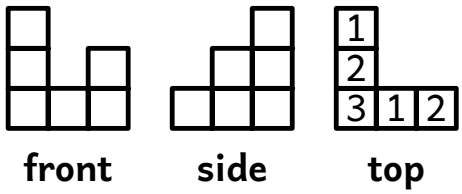
A stack of 9 cubes on dotted paper.

WORKED EXAMPLE

How many squares are in the front view of the stack in the picture?

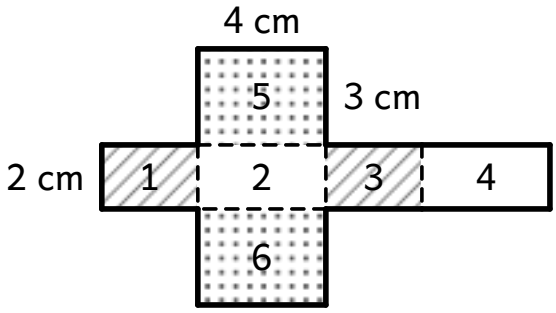
1. The front view has one column for each column of the stack. There are 3 columns.
2. Stand at the front. The left column has a tallest tower of 3 cubes.
3. The middle column has a tallest tower of 1 cube.
4. The right column has a tallest tower of 2 cubes.
5. The front view has 3 + 1 + 2 = 6 squares.

Answer: 6



Each view is a flat shape. The top view shows how many cubes stand on each square.

A **net** is a flat shape that folds up into a solid. The net of a cube is 6 squares joined along their sides. Not every set of 6 squares folds into a cube.



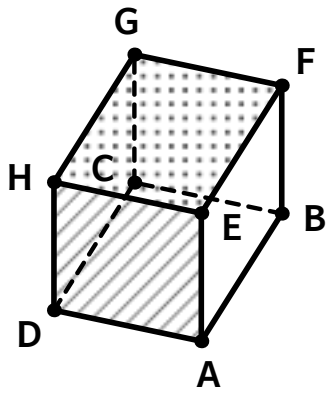
The net of a cuboid. Dashed lines are folds. Opposite faces have the same shading.

SEE IT THREE WAYS

With things

Take a small box, such as a tea box. Count its flat faces, then its edges, then its corners.

As a picture



In symbols

Faces + corners − edges = 6 + 8 − 12 = 2.

TRY IT

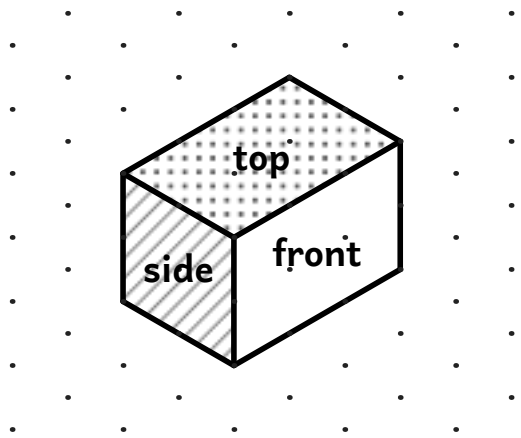
Count the parts of a box

You need: a small box, such as a tea box or a matchbox, a pencil, a sheet of paper.

1. Count the flat faces of the box. Write the number.
2. Count the edges. An edge is a line where two faces meet.
3. Count the corners.
4. Work out faces + corners − edges.

What you will see: A box has 6 faces, 8 corners and 12 edges. The answer to the sum is 2.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~To count the edges of a solid, count the lines you can see in the picture.~~

Why it feels right: A picture shows lines, so counting them seems enough.

The test that breaks it: Count the solid lines on the picture of the cube. You find 9. Now count the dashed lines too. There are 3 more, so the cube has 12 edges.

What is true: Edges at the back are drawn as dashed lines. Count the dashed lines too.

KEY WORDS

face
a flat surface of *a* solid.

edge
a line where two faces of *a* solid meet.

corner
a point where edges of *a* solid meet.

net
a flat shape that folds up into *a* solid.

view
the flat shape you see from the front, the side or the top.

BACK TO YOUR GUESS

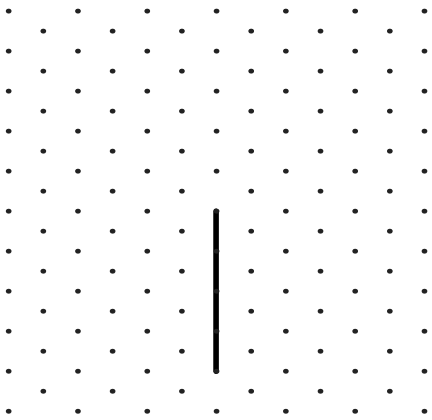
The answer is C: Yes. Faces plus corners take away edges is 2 for every solid like this.

For *a* cube, 6 + 8 − 12 = 2. The same sum gives 2 for *a* pyramid and for *a* prism. It holds for solids with flat faces and no hole through them. Adding the three numbers gives 26 for *a* cube, but it does not give 26 for other solids.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A cube has edges of 4 cm. Each step from dot to dot is 1 cm. How many steps long is each line on the dotted paper?



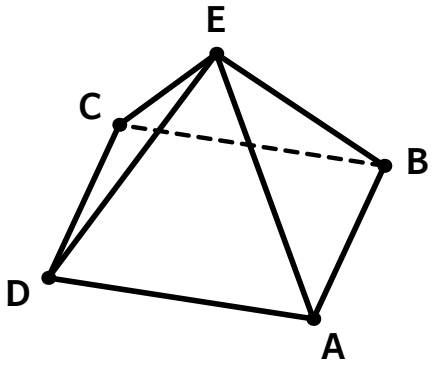
A 4

B 5

C 3

Answer at the back of the book.

2. How many edges has a square based pyramid?



A 5

B 8

C 4

Answer at the back of the book.

3. A triangular prism has 5 faces and 6 corners. How many edges has it? Use faces + corners − edges = 2.

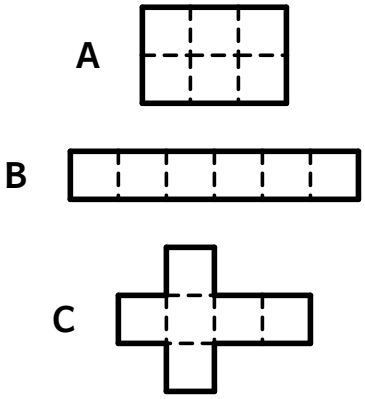
   A 13

   B 11

   C 9

   Answer at the back of the book.

4. Which of these is a net of a cube?



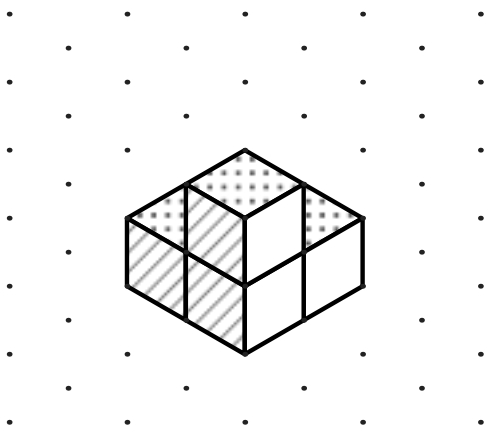
   A Net C

   B Net B

   C Net A

   Answer at the back of the book.

5. Look at the stack of cubes. The white faces on the right are the front. How many squares are in the front view?



A 4

B 3

C 2

Answer at the back of the book.

6. A solid has 7 faces and 15 edges. How many corners has it? Use faces + corners − edges = 2.

A 20

B 6

C 10

Answer at the back of the book.

7. A solid has 6 faces and 12 edges. How many corners has it?

Answer at the back of the book.

8. **From an earlier chapter**

A shape has turning symmetry of order 8. What is the smallest turn that fits it?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A solid has faces, edges and corners. Faces + corners − edges = 2. A net folds into a solid. A view is the flat shape you see from the front, side or top.

Draw: A cube on dotted paper, with the 9 edges you can see. Write 6 + 8 − 12 = 2 beside it.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

alternate angles
angles between the lines on opposite sides of the transversal, in a Z shape. Between parallel lines they are equal. M3.5.2

angle sum
the total of the angles in a shape. For a triangle it is 180°. M3.5.3

angles on a line
angles that fill a straight line. They add to 180°. M3.5.1

angles round a point
angles that fill a full turn. They add to 360°. M3.5.1

arc
a piece of a circle. M3.5.5

bisect
to cut exactly in half. The line that bisects an angle is the angle bisector. M3.5.5

centre of turning
the point a shape is turned about. M3.5.7

compasses
a tool that draws circles and arcs. It has a sharp point. M3.5.5

construction
a drawing made with a ruler and compasses, with the arcs left in. M3.5.5

corner
a point where edges of a solid meet. M3.5.8

corresponding angles
angles in the same place at each crossing, in an F shape. Between parallel lines they are equal. M3.5.2

diagonal
a line that joins two corners that are not next to each other. M3.5.4

edge
a line where two faces of a solid meet. M3.5.8

equilateral triangle
a triangle with three equal sides and three angles of 60°. M3.5.3

face
a flat surface of *a* solid. M3.5.8

image
the shape that *a* reflection makes. M3.5.6

isosceles triangle
a triangle with two equal sides and two equal angles. M3.5.3

line of symmetry
a line that cuts *a* shape into two halves that fold exactly onto each other. M3.5.6

mirror line
the line *a* shape is reflected in. M3.5.6

net
a flat shape that folds up into *a* solid. M3.5.8

order
how many times *a* shape fits on itself in one full turn. M3.5.7

parallel lines
straight lines that stay the same distance apart and never meet. M3.5.2

polygon
a flat shape with straight sides. With n sides, its angles add to $(n - 2) \times 180°$. M3.5.4

quadrilateral
a shape with four straight sides. Its angles add to 360°. M3.5.4

reflection
the shape you see on the other side of *a* mirror line. M3.5.6

regular polygon
a polygon with all sides equal and all angles equal. M3.5.4

right angled triangle
a triangle with one angle of 90°. M3.5.3

scalene triangle
a triangle with no equal sides. M3.5.3

transversal
a straight line that crosses two or more other lines. M3.5.2

turning symmetry
a shape has it when it fits on itself after *a* part turn. M3.5.7

vertex

the point where two lines meet. M3.5.1

vertically opposite angles

angles that face each other where two straight lines cross. They are equal. We also call them opposite angles. M3.5.1

view

the flat shape you see from the front, the side or the top. M3.5.8

Answers

M3.5.1. Angles on *a* line *and* round *a* point

1. **110°**

   The two angles fill *a* straight line, so they *a*dd to 180°. 180° − 70° = 110°.

   - **If you chose B:** You took 70° from 360°. A straight line is *a* half turn, 180°, not 360°. Go to M3.5.1.
   - **If you chose C:** You took 70° from 90°. Angles on *a* straight line *a*dd to 180°, not 90°. Go to M2.5.4.

2. **70°**

   The known *a*ngles *a*dd to 45° + 65° = 110°. Then 180° − 110° = 70°.

   - **If you chose A:** That is 45° + 65°, the two *a*ngles you know. T*a*ke it *a*way from 180° to find x. Go to M3.5.1.
   - **If you chose B:** You took only 45° from 180°. T*a*ke *a*way 65° *a*s well. Go to M3.5.1.

3. **220°**

   Angles round *a* point *a*dd to 360°. The known *a*ngles *a*dd to 80° + 60° = 140°. Then 360° − 140° = 220°.

   - **If you chose A:** That is 80° + 60°, the two *a*ngles you know. T*a*ke it *a*way from 360° to find x. Go to M3.5.1.
   - **If you chose C:** You used 180°, *a*s for *a* straight line. Angles round *a* point fill *a* full turn, which is 360°. Go to M3.5.1.

4. **52°**

   Angle c is opposite the 52° *a*ngle, *a*nd opposite *a*ngles *a*re equ*a*l. So c = 52°.

   - **If you chose B:** That is the *a*ngle next to 52°, on the same straight line. The opposite *a*ngle f*a*ces 52° *a*cross the vertex. Go to M3.5.1.
   - **If you chose C:** You took 52° from 90°. Opposite *a*ngles *a*re equ*a*l, so c is the same size *a*s 52°. Go to M3.5.1.

5. **Opposite angles where two straight lines cross**

   Two straight lines that cross make two pairs of opposite angles. Each pair is equal.

   - **If you chose A:** Side by side on a straight line they add to 180°. They are equal only when both are 90°. Go to M3.5.1.
   - **If you chose B:** Angles round a point add to 360°. They are not always equal. Go to M3.5.1.

6. **63°**

   180° − 117° = 63°.

7. **75°**

   85° + 140° + 60° = 285°. Then 360° − 285° = 75°.

M3.5.2. Angles and parallel lines

1. **70°**

   The angle x is in the same place at the other crossing. Corresponding angles between parallel lines are equal, so x = 70°.

   - **If you chose B:** That is 180° − 70°, the angle next to it. The angle in the same place is a corresponding angle, so it is equal to 70°. Go to M3.5.2.
   - **If you chose C:** You took 70° from 90°. Corresponding angles between parallel lines are equal. Go to M3.5.2.

2. **Alternate angles**

   Both angles are between the parallel lines, on opposite sides of the transversal. They make a Z shape, so they are alternate angles.

   - **If you chose A:** Corresponding angles are in the same place at each crossing, like an F. These are between the lines on opposite sides, like a Z. Go to M3.5.2.
   - **If you chose B:** Opposite angles face each other at one crossing. These two are at different crossings. Go to M3.5.1.

3. **125°**

   The two angles make a Z shape between the parallel lines. Alternate angles are equal, so y = 125°.

   - **If you chose A:** That is the angle next to 125° on a straight line. The alternate angle is equal to 125°. Go to M3.5.2.
   - **If you chose C:** You took 125° from 360°. Alternate angles are equal, so y is 125°. Go to M3.5.2.

4. **116°**

   At the top, the lower right angle is next to 64° on a straight line. So it is 180° − 64° = 116°. The angle x is in the same place at the bottom, so x = 116°.

   - **If you chose B:** The angle x is not in the same place as 64°. It is next to the corresponding angle, so it fills a straight line with it. Go to M3.5.2.
   - **If you chose C:** You took 64° from 90°. Find the angle in the same place as x at the top, then use 180°. Go to M3.5.1.

5. **Corresponding angles are equal.**

   Corresponding angles are in the same place at each crossing. Between parallel lines they are equal.

   - **If you chose A:** Alternate angles are equal. The angles that add to 180° are side by side on a straight line. Go to M3.5.2.
   - **If you chose B:** Only two sizes appear. Angles side by side add to 180°, so they are equal only when both are 90°. Go to M3.5.2.

6. **122°**

   The corresponding angle x is 58°. The angle next to x fills a straight line with it, so it is 180° − 58° = 122°.

7. **90°**

   38° + 52° = 90°. Then 180° − 90° = 90°.

M3.5.3. Triangles

1. **53°**

   55° + 72° = 127°. Then 180° − 127° = 53°.

   - **If you chose A:** That is 55° + 72°, the two angles you know. Take it away from 180° to find x. Go to M3.5.3.
   - **If you chose C:** You took 127° from 360°. The angles of a triangle add to 180°, not 360°. Go to M3.5.3.

2. **65°**

   The other two angles add to 180° − 50° = 130°. They are equal, so each is 130° ÷ 2 = 65°.

   - **If you chose A:** That is 180° − 50°. The two equal angles share it, so each is half of 130°. Go to M3.5.3.
   - **If you chose B:** The angle of 50° is at the top. The two base angles are equal to each other, but they are not 50°. Go to M3.5.3.

3. **Scalene triangle**

   All three sides have a different length, so the triangle is scalene.

   - **If you chose B:** An isosceles triangle has two sides of the same length. These three sides are all different. Go to M3.5.3.
   - **If you chose C:** An equilateral triangle has three equal sides. These sides are 4 cm, 5 cm and 6 cm. Go to M3.5.3.

4. **52°**

   The three angles add to 180°. 90° + 38° = 128°. Then 180° − 128° = 52°.

   - **If you chose A:** You took only 38° from 180°. Take away the right angle, 90°, as well. Go to M3.5.3.
   - **If you chose C:** The triangle already has one angle of 90°. The other two add to 90°, so the third is smaller. Go to M3.5.3.

5. **90°, 50° and 40°**

   90° + 50° + 40° = 180°. The other sets add to 190°.

   - **If you chose A:** These add to 190°. The angles of a triangle add to 180°. Go to M3.5.3.
   - **If you chose B:** These add to 190°. The angles of a triangle add to 180°. Go to M3.5.3.

6. **55°**

   48° + 77° = 125°. Then 180° − 125° = 55°.

7. **110°**

   The corresponding angle at the bottom is 70°. The angle y fills a straight line with it, so y = 180° − 70° = 110°.

M3.5.4. Angles in polygons

1. **110°**

   100° + 90° + 60° = 250°. Then 360° − 250° = 110°.

   - **If you chose A:** That is 100° + 90° + 60°, the three angles you know. Take it away from 360° to find x. Go to M3.5.4.
   - **If you chose C:** You may have used 180°, the angle sum of a triangle. The angles of a quadrilateral add to 360°. Go to M3.5.4.

2. **540°**

   A pentagon makes 5 − 2 = 3 triangles. 3 × 180° = 540°.

   - **If you chose A:** You multiplied 5 sides by 180°. Take 2 away first. A pentagon makes 3 triangles, not 5. Go to M3.5.4.
   - **If you chose B:** That is the angle sum of a quadrilateral. A pentagon has one more side, so one more triangle. Go to M3.5.4.

3. **108°**

   The angle sum is 540°. The 5 angles are equal, so each is 540° ÷ 5 = 108°.

   - **If you chose B:** That is the angle sum of all five angles. Share it between the 5 angles. Go to M3.5.4.
   - **If you chose C:** That is 360° ÷ 5. Use the angle sum of a pentagon, 540°, and share it between the 5 angles. Go to M3.5.4.

4. **5**

   A polygon with n sides makes n − 2 triangles. 7 − 2 = 5.

   - **If you chose A:** That is the number of sides. The corner cannot join itself or the 2 corners next to it. It draws 4 diagonals and makes 5 triangles. Go to M3.5.4.
   - **If you chose C:** You took away only 1. Take away 2: a polygon with n sides makes n − 2 triangles. Go to M3.5.4.

5. **120°**

   A hexagon makes 4 triangles, so its angle sum is 4 × 180° = 720°. The 6 angles are equal, so each is 720° ÷ 6 = 120°.

   - **If you chose A:** That is the sum of all six angles. Share it between the 6 corners. Go to M3.5.4.
   - **If you chose B:** 360° ÷ 6 is the turn at the centre, not the inside angle of the hexagon. Go to M3.5.4.

6. **1 260°**

   9 − 2 = 7 triangles. 7 × 180° = 1 260°.

7. **75°**

   The other two angles add to 180° − 30° = 150°. They are equal, so each is 150° ÷ 2 = 75°.

M3.5.5. Drawing with ruler and compasses

1. **4 cm**

   The base uses 5 cm and the arc from B uses 3 cm. The third side is 4 cm, so set the compasses to 4 cm.

   - **If you chose A:** That is the length of the base. The arc from C uses the third side. Go to M3.5.5.
   - **If you chose B:** That is the setting you already used for the arc from B. The arc from C uses the side that is left. Go to M3.5.5.

2. **No. The arcs do not meet.**

   3 cm and 2 cm together are 5 cm, which is less than 7 cm. The arcs cannot cross.

   - **If you chose A:** Not any three. The two short sides must be longer together than the long side. Here 3 + 2 is less than 7. Go to M3.5.5.
   - **If you chose C:** The ruler is not the problem. The sides are too short to reach each other, whatever tool you use. Go to M3.5.5.

3. **5 cm, 6 cm, 10 cm**

   5 + 6 = 11, which is more than 10. The other two sets have two sides that add to 7 or 4, not more than the third.

   - **If you chose B:** 2 + 2 is exactly 4. The arcs only touch on the base, so you get a flat line, not a triangle. Go to M3.5.5.
   - **If you chose C:** 3 + 4 is only 7, which is less than 8. The arcs would not meet. Go to M3.5.5.

4. **Draw a line at 50° from B. Set the compasses to 4 cm and draw an arc about B across the line.**

   The corner A is on the 50° line, and 4 cm from B. The arc about B at 4 cm crosses the line at A.

   - **If you chose A:** The 6 cm is the base BC, which is already drawn. The arc uses the side BA. Go to M3.5.5.
   - **If you chose B:** That draws a triangle from three sides. Here you know an angle, so you draw a line at 50° from B first. Go to M3.5.5.

5. **50°**

   The line through R cuts the angle exactly in half. 100° ÷ 2 = 50°.

   - **If you chose A:** That is double the angle. Cutting an angle in half makes the parts smaller, not bigger. Go to M3.5.5.
   - **If you chose C:** That is a quarter of 100°. Cutting in half means dividing by 2. Go to M3.5.5.

6. **42°**

   Each half is 84° ÷ 2 = 42°.

7. **62°**

   The three angles add to 180°. 90° + 28° = 118°. Then 180° − 118° = 62°.

M3.5.6. Line symmetry and reflection

1. **4**

   A square folds onto itself across the middle two ways and along each diagonal. That is 2 + 2 = 4 lines.

   - **If you chose A:** You found the two lines through the middle of opposite sides. There are two more, along the diagonals. Go to M3.5.6.
   - **If you chose B:** There is more than one fold that makes the halves match. Try the middle of the sides and the diagonals. Go to M3.5.6.

2. **Parallelogram**

   No fold makes the two halves of a parallelogram lie on top of each other. The rectangle has 2 lines and the kite has 1.

   - **If you chose A:** A rectangle folds onto itself across the middle, top to bottom and left to right. Go to M3.5.6.
   - **If you chose C:** A kite has one line of symmetry, from the top corner to the bottom corner. Go to M3.5.6.

3. **(8, 3)**

   A is 5 − 2 = 3 squares left of the mirror line. A' is 3 squares to the right, at 5 + 3 = 8. The height stays 3.

   - **If you chose B:** The mirror line is upright, so only the first number changes. The second number stays 3. Go to M3.5.6.
   - **If you chose C:** A is 3 squares from the mirror line, and A' is 3 squares on the other side. 5 + 3 is 8, not 7. Go to M3.5.6.

4. **6**

   Three lines join opposite corners and three join the middles of opposite sides. 3 + 3 = 6.

   - **If you chose A:** You found the lines through opposite corners. There are also three lines through the middles of opposite sides. Go to M3.5.6.
   - **If you chose B:** That is two for each side. A line of symmetry goes right across the shape, so each line is counted once. Go to M3.5.6.

5. **(1, 2)**

   P is 4 − 3 = 1 square above the mirror line. P' is 1 square below, at 3 − 1 = 2. The first number stays 1.

   - **If you chose A:** You moved P up by 1 again. The image is on the other side of the mirror line, below it. Go to M3.5.6.
   - **If you chose C:** That is on the mirror line itself. P is 1 square above it, so P' is 1 square below: 3 − 1 = 2. Go to M3.5.6.

6. **7**

   A regular polygon has one line of symmetry for each side, so 7 sides give 7 lines.

7. **135°**

   An octagon makes 8 − 2 = 6 triangles, so the angle sum is 6 × 180° = 1 080°. Then 1 080° ÷ 8 = 135°.

M3.5.7. Turning symmetry

1. **3**

   The triangle fits on itself after 120°, 240° and 360°. That is 3 times, so the order is 3.

   - **If you chose A:** Order 1 means it fits only after the full turn. This triangle also fits after a turn of 120°. Go to M3.5.7.
   - **If you chose B:** There are 3 corners, and each turn moves a corner to the next corner. So there are 3 turns, not 6. Go to M3.5.7.

2. **Parallelogram**

   A parallelogram fits on itself after a half turn, so its order is 2. No fold makes its halves match, so it has no line of symmetry.

   - **If you chose A:** A rectangle has order 2, but it also has 2 lines of symmetry. Go to M3.5.6.
   - **If you chose C:** An isosceles triangle has 1 line of symmetry and its order is 1. Go to M3.5.7.

3. **72°**

   The 5 turns make one full turn of 360°. Each turn is 360° ÷ 5 = 72°.

   - **If you chose B:** That is the turn for order 4. Divide 360° by 5. Go to M3.5.7.
   - **If you chose C:** That is the turn for order 6. Divide 360° by 5. Go to M3.5.7.

4. **90°**

   Four copies make one full turn of 360°. Each turn is 360° ÷ 4 = 90°.

   - **If you chose A:** That would give 8 copies in a full turn. For 4 copies divide 360° by 4. Go to M3.5.7.
   - **If you chose B:** That is a half turn. It would give only 2 copies in a full turn. Go to M3.5.7.

5. **Isosceles triangle**

   An isosceles triangle has 1 line of symmetry. It fits on itself only after the full turn, so its order is 1.

   - **If you chose A:** A square has 4 lines of symmetry and also turning symmetry of order 4. Go to M3.5.7.
   - **If you chose C:** A parallelogram has turning symmetry of order 2 but no line of symmetry. This question asks the other way round. Go to M3.5.7.

6. **6**

   A regular polygon fits on itself once for each of its corners in a full turn. 6 sides give order 6.

7. **(10, 3)**

   The point is 6 − 2 = 4 squares left of the mirror line. The image is 4 squares right, at 6 + 4 = 10. The height stays 3.

M3.5.8. Solids, nets and views

1. **4**

   Each step is 1 cm, so an edge of 4 cm is 4 ÷ 1 = 4 steps long. It passes over 5 dots, but the steps are what count.

   - **If you chose B:** You counted the dots. A line that is 4 cm long goes over 5 dots but is only 4 steps long. Go to M3.5.8.
   - **If you chose C:** That is the size of the cube drawn in the chapter. This cube has edges of 4 cm. Go to M3.5.8.

2. **8**

   The base has 4 edges and 4 more edges go up to the top corner. 4 + 4 = 8.

   - **If you chose A:** That is the number of faces, and also the number of corners. Count the lines: 4 on the base and 4 up to the top. Go to M3.5.8.
   - **If you chose C:** That is the edges of the base only. Add the 4 edges that go up to the top corner. Go to M3.5.8.

3. **9**

   Faces + corners − edges = 2, so edges = faces + corners − 2. 5 + 6 − 2 = 9.

   - **If you chose A:** You added the 2. The rule says faces + corners − edges = 2, so edges = faces + corners − 2. Go to M3.5.8.
   - **If you chose B:** That is 5 + 6. Take away the 2 as well. Go to M3.5.8.

4. **Net C**

   Fold the four squares in the row round the cube. The square above and the square below fold on to the top and the bottom. Every face is covered once.

   - **If you chose B:** A straight row of 6 squares wraps round the cube. The fifth square lands on the first, and the sixth on the second. Go to M3.5.8.
   - **If you chose C:** A block of 2 rows of 3 squares will not fold. Two of the squares would land on the same face of the cube. Go to M3.5.8.

5. **3**

   The left column of the front view is as tall as its tallest tower, 2 squares. The right column has 1 square. 2 + 1 = 3 squares.

   - **If you chose A:** That counts the cube behind the tower of 2 as a third column, or counts every cube. The front is the white faces on the right. The cube behind adds no square. Go to M3.5.8.
   - **If you chose C:** That is the height of the tallest tower. The front view also has a column for the tower of 1 cube. Go to M3.5.8.

6. **10**

   Faces + corners − edges = 2, so corners = 2 + edges − faces. 2 + 15 − 7 = 10.

   - **If you chose A:** You added 7 and 15 and took away 2. Put the rule in order: corners = 2 + edges − faces. Go to M3.5.8.
   - **If you chose B:** You took 2 away instead of adding it. Corners = 2 + edges − faces. Go to M3.5.8.

7. **8**

   Corners = 2 + edges − faces = 2 + 12 − 6 = 8.

8. **45°**

   The 8 turns make one full turn of 360°. Each turn is 360° ÷ 8 = 45°.

About this book

For learners who can measure an angle with a protractor and know that a right angle is 90°. In many schools this fits children of about 11 to 13, and older learners who never met angle rules can start here too. Each chapter takes one or two short sessions. A learner needs a ruler, a protractor, a pencil and, for one chapter, a pair of compasses with a sharp point, so an adult should be near. Every exercise has its answer and the reason for it. Each chapter also has a short page for the adult, with three questions to ask.

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