●●●○○ Level 3 · Mathematics M3.2

Long multiplication and division

Multiply and divide big numbers on paper, and check answers with estimates.

Multiply and divide in your head, then on paper with the grid method, long multiplication, short division and long division. Learn the order of operations, check answers with estimates, and choose the right operation for a problem.

YOU ARE READY FOR THIS BOOK IF

You can work out 23 × 4 and 84 ÷ 4 on paper.

This book	**M3.2 Long multiplication and division**
One step down	M2.3 Times tables and sharing

Contents

M3.2.1 Multiplying and dividing in your head

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M3.2.7 Estimate, then check

M3.2.8 Choosing what to do

Key words, A to Z

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M3.2.1 CHAPTER 1

Multiplying and dividing in your head

How can I work out 25 × 16 without writing it down?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Work out 25 × 10 = 250, then add 6. The answer is 256.
- ☐ **B** Double 25 and halve 16, again and again. The answer is 400.
- ☐ **C** You cannot do it without paper.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.8. If multiplying by 10 and 100 is new to you, read this first.
- Level 2, Big numbers and place value, M2.1.2. If the place of each digit in a big number is not clear, read this first.

BY THE END YOU CAN

- Multiply and divide by 10, 100 and 1 000
- Use doubling and halving to work out 25 × 16
- Multiply multiples of ten, like 30 × 40

Kofi plants 16 rows of seeds. Each row has 25 seeds. How many seeds is that? You can find out in your head, with three short tricks.

The first trick is about place value. When you multiply by 10, every digit moves one place to the left. A 0 fills the empty ones place.

DRAW IT FIRST

Move the digits to multiply by 10 and 100

35

thousands	hundreds	tens	ones
		3	5

1 Write 35 in the chart. The 3 is in the tens place and the 5 is in the ones place.

35 × 10 = 350

thousands	hundreds	tens	ones
	3	5	0

2 Multiply by 10. Both digits move one place to the left, and a 0 fills the ones place. The answer is 350.

35 × 100 = 3 500

thousands	hundreds	tens	ones
3	5	0	0

3 Multiply by 100. Both digits move two places to the left, and two zeros fill the gap. The answer is 3 500.

To divide by 10, 100 or 1 000, move the digits the other way, to the right. 4 500 ÷ 100 = 45, because each digit moves two places to the right.

4 500

thousands	hundreds	tens	ones
4	5	0	0

4 500 ÷ 100 = 45

thousands	hundreds	tens	ones
		4	5

Dividing by 100 moves each digit two places to the right.

WORKED EXAMPLE

Work out 4 500 ÷ 100.

1. Dividing by 100 moves every digit two places to the right.
2. The 4 moves from thousands to tens. The 5 moves from hundreds to ones.
3. The two zeros move off the end and are not needed.
4. So 4 500 ÷ 100 = 45.

Answer: 45

The second trick is **doubling** and **halving**. To double a number is to make it twice as big. To halve it is to cut it into two equal parts.

first	second
25	16
50	8
100	4
200	2

Each row doubles the first number and halves the second. Every row multiplies to 400.

WORKED EXAMPLE

Work out 25 × 16 by doubling and halving.

1. Write the calculation as 25 × 16.
2. Halve 16 to make 8, and double 25 to make 50.
3. Do it again. Halve 8 to make 4, and double 50 to make 100.
4. Now the calculation is 100 × 4. You can work it out at once: 100 × 4 = 400.
5. So 25 × 16 = 400.

Answer: 400

The third trick is for **multiples of ten**, such as 30 and 40. Leave out the zeros at the end of each number. Multiply the other digits. Then put those zeros back.

SEE IT THREE WAYS

With things

A farmer plants 30 rows of 40 trees. Think of 30 as 3 tens and 40 as 4 tens.

As a picture

	40
30	1 200

total 1 200

not to scale

In symbols

3 × 4 = 12, so 30 × 40 = 1 200. Two zeros go back after the 12.

WORKED EXAMPLE

Work out 30 × 40.

1. 30 × 40 means 3 tens × 4 tens. First work out 3 × 4 = 12.
2. Each number had one zero, so the answer needs two zeros after the 12.
3. Put them back. 12 with two zeros after it is 1 200.
4. Check another way. 30 × 4 = 120.
5. Then multiply by 10. 120 × 10 = 1 200.

Answer: 1 200

TRY IT

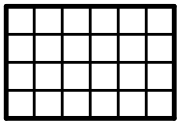
Double one, halve the other

You need: 24 small stones, beans or buttons.

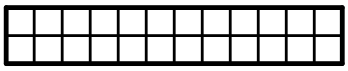
1. Lay the stones in 4 rows of 6. Count them.
2. Move row 2 to the end of row 1. Move row 4 to the end of row 3. Now you have 2 rows of 12.
3. Count again, without moving a stone.

What you will see: There are 24 both times. You halved the number of rows and doubled the length of each row. So 4 × 6 = 2 × 12.

If you cannot try it, this is what happens.



4 rows of 6



2 rows of 12

A WRONG IDEA MANY PEOPLE HAVE

~~To keep the answer the same, double both numbers.~~

Why it feels right: Doubling and halving are both fair changes, so doubling twice feels fair too.

The test that breaks it: Try it on 25 × 16 = 400. Double both and you get 50 × 32. That is 1 600, which is four times 400.

What is true: Double one number and halve the other. If you double both, the answer becomes four times as big.

KEY WORDS

doubling
making a number twice as big.

halving
cutting a number into two equal parts.

multiples of ten
numbers you get by multiplying a whole number by 10, such as 30, 40 and 700.

BACK TO YOUR GUESS

The answer is B: Double 25 and halve 16, again and again. The answer is 400.

Doubling one number and halving the other keeps the answer the same. 25 × 16 = 50 × 8 = 100 × 4 = 400. The first idea adds only 6, but you need 6 more lots of 25.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Work out 67 × 100.

   Answer at the back of the book.

2. Work out 8 000 ÷ 100.

   Answer at the back of the book.

3. Which calculation has the same answer as 25 × 16?

   A 50 × 8

   B 50 × 32

   C 25 × 8

   D 12 × 32

   Answer at the back of the book.

4. How do the digits of 62 move when you multiply 62 by 1 000?

   A The digits stay still, because 1 000 is added to make 1 062.

   B One place to the left, so the answer is 620.

   C Three places to the left, so the answer is 62 000.

   D Three places to the right, so the answer is smaller.

   Answer at the back of the book.

5. The chart shows 4 500. What is 4 500 ÷ 10?

thousands	hundreds	tens	ones
4	5	0	0

   A 4 500

   B 45

   C 45 000

   D 450

   Answer at the back of the book.

6. Maya works out 35 × 12. She doubles 35 to make 70 and halves 12 to make 6. What is 70 × 6?

   A 840

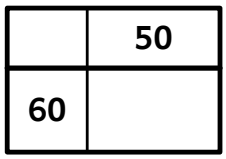
   B 420

   C 210

   D 76

   Answer at the back of the book.

7. The grid shows the pieces 50 and 60. What goes in the box?



not to scale

A 3 000

B 300

C 30 000

D 110

Answer at the back of the book.

8. Work out 60 × 70.

Answer at the back of the book.

9. Use doubling and halving to work out 35 × 14.

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

To multiply by 10, 100 or 1 000, move every digit left. To divide, move them right. Double one number and halve the other. For 30 × 40, work out 3 × 4, then put back two zeros.

Draw: Draw a place value chart with 35 in it, and under it 350. Draw an arrow from each digit to its new place.

M3.2.2 CHAPTER 2

Short multiplication

How do I work out 4 × 326 on paper?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Multiply each digit by 4 and write the answers side by side: 12, 8 and 24.
- ☐ **B** Multiply only the hundreds: 4 × 300 = 1 200.
- ☐ **C** Split 326 into 300, 20 and 6. Multiply each part by 4, then add. The answer is 1 304.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.8. If 23 × 4 on paper is new to you, read this first.
- Level 2, Big numbers and place value, M2.1.1. If you cannot say what each digit of 326 is worth, read this first.

BY THE END YOU CAN

- Multiply a 3 or 4 digit number by a 1 digit number
- Use the grid method to show what each part means

A lorry makes 4 trips. Each trip carries 326 bricks. How many bricks is that? You could add 326 four times. Multiplying is quicker.

The **grid method** splits 326 into 300 + 20 + 6. Each part is multiplied by 4 in its own box. Then you add the boxes.

DRAW IT FIRST

The grid method for 4 × 326

	300	20	6
4			

not to scale

1 Split 326 into 300, 20 and 6. Write them across the top. Write 4 down the side.

	300	20	6
4	1 200	80	24

not to scale

2 Multiply each part by 4. 4 × 300 = 1 200, 4 × 20 = 80 and 4 × 6 = 24.

	300	20	6
4	1 200	80	24

total 1 304

not to scale

3 Add the three boxes. 1 200 + 80 + 24 = 1 304.

The written method is the grid method made short. Start with the ones. When a product has two digits, write the last digit and **carry** the rest to the next place.

$$\begin{array}{r} 3\ 2\ 6 \\ \times\quad\ \ 4 \\ \hline 1\ 3\ 0\ 4 \end{array}$$

1 2

The small 2 and 1 are the carried digits.

WORKED EXAMPLE

Work out 326 × 4 in columns.

1. Ones: write 4 and carry 2, because 4 × 6 = 24.
2. Tens: write 0 and carry 1, because 4 × 2 = 8 and 8 + 2 = 10.
3. Hundreds: write 13, because 4 × 3 = 12 and 12 + 1 = 13.
4. Read the answer from the left: 1 304.

Answer: 1 304

WORKED EXAMPLE

Work out 2 748 × 6 in columns.

1. Ones: write 8 and carry 4, because 8 × 6 = 48.
2. Tens: write 8 and carry 2, because 4 × 6 = 24 and 24 + 4 = 28.
3. Hundreds: write 4 and carry 4, because 7 × 6 = 42 and 42 + 2 = 44.
4. Thousands: write 16, because 2 × 6 = 12 and 12 + 4 = 16.
5. Read the answer from the left: 16 488.

Answer: 16 488

WORKED EXAMPLE

Work out 4 308 × 5 with the grid method.

1. Split 4 308 into 4 000 + 300 + 8. There are no tens, so there is no tens box.
2. 5 × 4 000 = 20 000.
3. 5 × 300 = 1 500.
4. 5 × 8 = 40.
5. Add the boxes. 20 000 + 1 500 + 40 = 21 540.

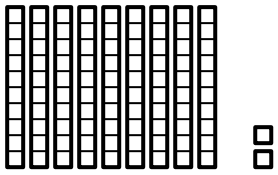
Answer: 21 540

SEE IT THREE WAYS

With things

Make 4 piles of blocks. Each pile has 2 rods of ten and 3 single cubes. Push the piles together. Trade 10 cubes for 1 rod.

As a picture



4 × 23 = 92

In symbols

4 × 20 = 80 and 4 × 3 = 12. 80 + 12 = 92.

TRY IT

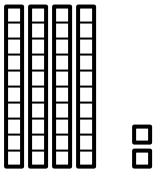
Make 3 × 14 with sticks

You need: 42 short sticks: 3 bundles of ten tied with string, and 12 loose sticks, string.

1. Make 3 groups. Put 1 bundle of ten and 4 single sticks in each group.
2. Push the 3 groups together. Count the bundles and the single sticks.
3. You will have 12 single sticks. Tie 10 of them into a new bundle.

What you will see: You have 4 bundles of ten and 2 single sticks. That is 42. So 3 × 14 = 42.

If you cannot try it, this is what happens.



3 × 14 = 42

A WRONG IDEA MANY PEOPLE HAVE

~~When you carry, add the carried number to the digit first, then multiply.~~

Why it feels right: In adding, you put the carried 1 with the next column before you do anything else.

The test that breaks it: Try 326 × 4 the wrong way in every column. After carrying 2, add it to the 2 to make 4. Then 4 × 4 = 16, so write 6 and carry 1. In the hundreds, (3 + 1) × 4 = 16. You end with 1 664. The grid method gives 1 304.

What is true: Multiply the digit first. Then add the carried number to the product.

KEY WORDS

grid method
splitting a number by place value and multiplying each part in its own box.

carry
to move the tens of a product to the next place on the left.

BACK TO YOUR GUESS

The answer is C: Split 326 into 300, 20 and 6. Multiply each part by 4, then add. The answer is 1 304.

4 × 326 = 1 304. Each part is multiplied: 4 × 300 = 1 200, 4 × 20 = 80 and 4 × 6 = 24. Then you add them. The 12, 8 and 24 are right, but they are worth different amounts.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Use the grid method to work out 3 × 245.

   Answer at the back of the book.

2. Build the answer to 4 × 23 with blocks.

   Answer at the back of the book.

3. What is 3 × 418?

   A 12 324

   B 1 234

   C 1 294

   D 1 254

   Answer at the back of the book.

4. In the grid for 3 × 245, what goes in the box under the 40?

	200	**40**	**5**
3			

not to scale

   A 120

   B 12

   C 43

   D 600

   Answer at the back of the book.

5. Which three numbers does the grid method add up for 4 × 326?

   A 12, 8 and 24

   B 1 200, 80 and 24

   C 300, 20 and 6

   D 1 200, 80 and 6

   Answer at the back of the book.

6. The calculation is 2 748 × 6. The first step is 8 × 6 = 48. What do you write?

$$\begin{array}{r} 2\,7\,4\,8 \\ \times \qquad 6 \\ \hline \end{array}$$

A Write 8 and forget the 4.

B Write 48 under the 8.

C Write 4 and carry 8.

D Write 8 and carry 4.

Answer at the back of the book.

7. What is 1 523 × 4?

A 6 082

B 4 092

C 6 092

D 420 812

Answer at the back of the book.

8. Use the grid method to work out 7 × 3 062.

Answer at the back of the book.

9. **From an earlier chapter**

Work out 400 × 30.

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Grid method: split into hundreds, tens and ones, multiply each part, add the boxes. In columns: start at the ones. Multiply, add the carry, write the last digit, carry the rest.

Draw: Draw a grid for 4 × 326 with 300, 20 and 6 on top and 4 at the side. Write each product in its box and add them.

M3.2.3 CHAPTER 3

Long multiplication

How do I work out 346 × 52 on paper?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 346 × 5 = 1 730 and 346 × 2 = 692. Add them to get 2 422.
- ☐ **B** Work out 346 × 50 = 17 300 and stop.
- ☐ **C** Multiply 346 by the 2, then by the 50, and add the two rows. The answer is 17 992.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.8. If 23 × 4 on paper is new to you, read this first.
- Level 2, Adding and taking away big numbers, M2.2.3. If adding two rows in columns is not firm yet, read this first.

BY THE END YOU CAN

- Multiply a 3 digit number by a 2 digit number
- Explain why the second row starts with a zero

A shop has 52 boxes. Each box holds 346 pencils. How many pencils is that? You know how to multiply by one digit. Now the multiplier has two digits.

52 means 50 + 2. So multiply 346 by 2, then multiply 346 by 50, then add. First look at it as a grid.

DRAW IT FIRST

The grid for 346 × 52

	300	40	6
50			
2			

not to scale

1 Split both numbers by place. 346 is 300, 40 and 6. 52 is 50 and 2.

	300	40	6
50	15 000	2 000	300
2	600	80	12

not to scale

2 Multiply each pair. There are six boxes, such as 50 × 300 = 15 000.

	300	40	6
50	15 000	2 000	300
2	600	80	12

total 17 992

not to scale

3 Add all six boxes. The total is 17 992.

Long multiplication does the same job with two rows. Each row is a **partial product**. It is part of the answer, not the whole answer.

DRAW IT FIRST

Long multiplication, row by row

```
   3 4 6
×    5 2
-------
```

1 Write 346 above 52, with each digit in its own column. Draw a line.

```
   3 4 6
×    5 2
-------
   6 9 2
```

2 Row 1: multiply 346 by the 2 ones. 346 × 2 = 692.

```
   3 4 6
×    5 2
-------
   6 9 2
+ 1 7 3 0 0
```

3 Row 2: write a 0 first. Then multiply 346 by the 5 tens. 346 × 5 = 1 730, so the row reads 17 300.

```
   3 4 6
×    5 2
-------
   6 9 2
+ 1 7 3 0 0
-----------
 1 7 9 9 2
```

4 Add the rows. 692 + 17 300 = 17 992.

Why does row 2 start with a 0? The 5 is not 5 ones. It is 5 tens. So row 2 is 346 × 50. The 0 is a **placeholder**. It keeps the ones place empty, so each digit lands in the right place.

WORKED EXAMPLE

Work out 346 × 52.

1. Row 1: multiply 346 by the 2 ones. 346 × 2 = 692.
2. Row 2: write *a* 0 in the ones place first. This row is for tens.
3. Multiply 346 by the 5 tens. 346 × 5 = 1 730. Write 1 730 in front of the 0, so the row reads 17 300.
4. Add the two rows. 692 + 17 300 = 17 992.

Answer: 17 992

WORKED EXAMPLE

Work out 128 × 37.

1. Row 1: 128 × 7 = 896.
2. Row 2: start with *a* 0 in the ones place.
3. Then 128 × 3 = 384, so row 2 reads 3 840.
4. Add the rows. 896 + 3 840 = 4 736.

Answer: 4 736

WORKED EXAMPLE

Work out 408 × 63.

1. Row 1: 3 × 408 = 1 224.
2. Row 2 is for the 6 tens, so it starts with *a* 0 in the ones place.
3. Then 6 × 408 = 2 448.
4. Put the 0 after it. Row 2 reads 24 480.
5. Add the rows. 1 224 + 24 480 = 25 704.

Answer: 25 704

SEE IT THREE WAYS

With things

Make 23 rows of stones with 14 stones in each row. Think of it as 20 rows and 3 rows.

As a picture

	10	4
20	200	80
3	30	12

total 322

not to scale

In symbols

23 × 14 = 20 × 14 + 3 × 14 = 280 + 42 = 322.

TRY IT

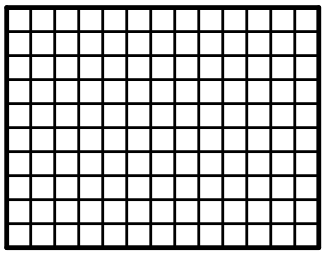
Cut a rectangle in two

You need: squared paper, or a page where you draw 13 squares by 12 squares, a pencil.

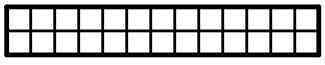
1. Draw a rectangle 13 squares wide and 12 squares tall.
2. Draw a line to cut off the top 10 rows. The bottom 2 rows are left.
3. Count the squares in each block by multiplying. Add the two counts.

What you will see: The top block has 10 × 13 = 130 squares. The bottom block has 2 × 13 = 26. Together they make 156, so 12 × 13 = 156.

If you cannot try it, this is what happens.



10 rows of 13 = 130



2 rows of 13 = 26

A WRONG IDEA MANY PEOPLE HAVE

~~The 0 on the second row is only for neatness. You can leave it out.~~

Why it feels right: The row is already lined up under the first row, so the 0 looks like an extra.

The test that breaks it: Leave the 0 out of 346 × 52. Row 2 becomes 1 730, and 692 + 1 730 = 2 422. But 350 × 50 is about 17 500, so 2 422 is far too small.

What is true: The 5 is worth 50, so the row is 346 × 50. The 0 holds the ones place and moves the other digits to the left.

KEY WORDS

partial product
one row of a long multiplication, a part of the full answer.

placeholder
a 0 that holds a place empty, so the other digits stay in the right places.

BACK TO YOUR GUESS

The answer is C: Multiply 346 by the 2, then by the 50, and add the two rows. The answer is 17 992.

The 5 in 52 is worth 50, so one row is 346 × 50 = 17 300. The other row is 346 × 2 = 692. The two rows add to 17 992. The first idea treats the 5 as 5 ones. The third forgets the 2.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Work out 24 × 13 on paper.

   Answer at the back of the book.

2. A library has 38 shelves. Each shelf holds 245 books. How many books is that?

   Answer at the back of the book.

3. What is 245 × 36?

   A 7 350

   B 2 205

   C 1 470

   D 8 820

   Answer at the back of the book.

4. Why does the second row of 346 × 52 start with a 0?

$$\begin{array}{rr} & 3\,4\,6 \\ \times & 5\,2 \\ \hline & 6\,9\,2 \\ + \; 1\,7\,3\,0 & \boxed{0} \end{array}$$

   A The digit is worth tens. The row is ten times as big, and the zero holds the ones place.

   B It only helps the row line up neatly. It has no value.

   C Because any number times 5 ends in a zero.

   D Because 346 × 2 ends in a zero.

   Answer at the back of the book.

5. What are the two rows of 215 × 43?

$$\begin{array}{rr} & 2\ 1\ 5 \\ \times & 4\ 3 \\ \hline \end{array}$$

A 645 and 860

B 645 and 8 600

C 860 and 645

D 645 and 86 000

Answer at the back of the book.

6. Joseph works out 128 × 37. He writes 896 and 384, and adds them to get 1 280. What went wrong?

A Nothing. The answer is right.

B He added the two rows wrongly.

C Row 1 should have been 128 × 3.

D The second row should be 3 840, not 384. He left out the zero.

Answer at the back of the book.

7. The grid shows 346 × 52. Which boxes add up to row 2, the row for the 50?

	300	**40**	**6**
50	15 000	2 000	300
2	600	80	12

not to scale

A 15 000, 2 000 and 300

B 600, 80 and 12

C 15 000 and 600

D 300, 40 and 6

Answer at the back of the book.

8. Work out 307 × 48.

   Answer at the back of the book.

9. **From an earlier chapter**

   Use the grid method to work out 4 × 1 263.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Multiply by the ones. Then write *a* 0 and multiply by the tens. Add the two rows. The 0 is there because the second row is worth tens.

Draw: Draw 346 × 52 in columns with the two rows. Ring the 0 at the end of row 2 and write "row for the tens" beside it.

M3.2.4 CHAPTER 4

Short division

How do I divide 5 217 by 7 on paper?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Divide each digit by 7 on its own, starting from the right.
- ☐ **B** Divide the 5 by 7, and then do the same for 2, 1 and 7. The answer is 0, 0, 0 and 1.
- ☐ **C** Start from the left. Say how many 7s go into each number, and carry what is left over. The answer is 745 with 2 left over.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.7. If a remainder is a new word, read this first.
- Level 2, Times tables and sharing, M2.3.3. If you are not sure what 12 ÷ 3 means, read this first.

BY THE END YOU CAN

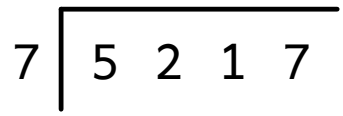
- Divide a 4 digit number by a 1 digit number
- Write a remainder as a whole number or as a fraction

Leila shares 5 217 beads equally between 7 bags. How many beads go in each bag? How many are left over?

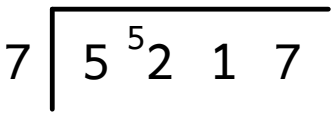
This is **short division**. It is also called the bus stop method, because of the shape of the bracket. The number you divide by is the **divisor**. Work from the left. When a number does not divide exactly, carry what is left over to the next digit.

DRAW IT FIRST

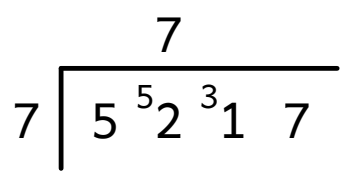
Short division, one digit at a time



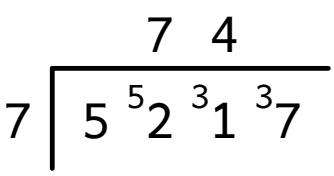
1 Write the divisor 7 outside the bracket. Write 5 217 inside it, one digit in each place.



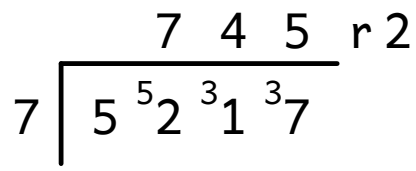
2 7 goes into 5 no times. Write nothing above. Carry the 5, so the next number is 52.



3 7 goes into 52 seven times, because 7 × 7 = 49. Write 7 above the 2. Carry the 3, so the next number is 31.



4 7 goes into 31 four times, because 7 × 4 = 28. Write 4 above the 1. Carry the 3, so the next number is 37.



5 7 goes into 37 five times, because 7 × 5 = 35. Write 5 above the 7. The last leftover is 2.

The 2 left at the end is the **remainder**. Each bag gets 745 beads, and 2 beads are left over. You write it as 745 r 2.

You can also write the remainder over the divisor, as a fraction. Suppose the 2 left over could be cut up. Shared by the 7 bags, each bag would get $^2/_7$ more. So the answer is 745 $^2/_7$. Use a fraction only when the thing can be cut, like rope or water. Beads cannot be cut, so for beads 745 r 2 is the honest answer.

$$7 \overline{)\,5^{5}2^{3}1^{3}7} \quad 7\,4\,5\ \tfrac{2}{7}$$

The same calculation, with the remainder written as a fraction.

WORKED EXAMPLE

Work out 4 806 ÷ 6.

1. 6 goes into 4 no times. Carry the 4, so the next number is 48.
2. 6 goes into 48 eight times, because 6 × 8 = 48. Nothing is left over.
3. The next digit is 0. 6 goes into 0 no times, so write 0 above it.
4. 6 goes into 6 once, because 6 × 1 = 6. The answer is 801.

Answer: 801

WORKED EXAMPLE

Work out 3 458 ÷ 5.

1. 5 goes into 3 no times. Carry the 3, so the next number is 34.
2. 5 goes into 34 six times, because 5 × 6 = 30. The leftover is 34 − 30 = 4.
3. Carry the 4, so the next number is 45. 5 goes into 45 nine times, because 5 × 9 = 45. Nothing is left over.
4. 5 goes into 8 once, because 5 × 1 = 5. The leftover is 8 − 5 = 3.
5. The answer is 691 with remainder 3.

Answer: 691 remainder 3

WORKED EXAMPLE

Work out 9 175 ÷ 6. Write the remainder over the divisor, as a fraction.

1. 6 goes into 9 once, because 6 × 1 = 6. The leftover is 9 − 6 = 3.
2. Carry the 3, so the next number is 31. 6 goes into 31 five times, because 6 × 5 = 30. The leftover is 31 − 30 = 1.
3. Carry the 1, so the next number is 17. 6 goes into 17 twice, because 6 × 2 = 12. The leftover is 17 − 12 = 5.
4. Carry the 5, so the next number is 55. 6 goes into 55 nine times, because 6 × 9 = 54. The leftover is 55 − 54 = 1.
5. The remainder 1 over the divisor 6 is $^1/_6$. The answer is 1 529 $^1/_6$.

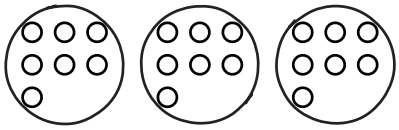
Answer: 1 529 $^1/_6$

SEE IT THREE WAYS

With things

Share 23 stones between 3 people. Give each person one stone at a time. Stop when you cannot give one to everyone.

As a picture



In symbols

23 ÷ 3 = 7 remainder 2. If the 2 stones could be cut into thirds, each person would get 7 and $^2/_3$.

TRY IT

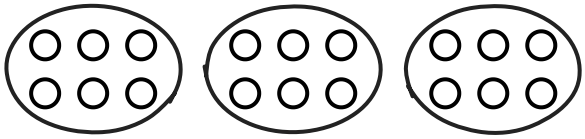
Share 20 stones between 3 cups

You need: 20 small stones, beans or buttons, 3 cups or bowls.

1. Put one stone in each cup. Keep going, one at a time.
2. Stop when there are not enough stones to give one to every cup.
3. Count the stones in each cup. Count the stones left in your hand.

What you will see: Each cup has 6 stones, and 2 stones are left over. So 20 ÷ 3 = 6 remainder 2.

If you cannot try it, this is what happens.



 2 left over

A WRONG IDEA MANY PEOPLE HAVE

~~Start dividing from the right, as you do when you add.~~

Why it feels right: Adding, taking away and multiplying all start with the ones place.

The test that breaks it: Try 72 ÷ 3 from the right. 2 ÷ 3 is 0, with 2 left over. There is no place to the right to carry the 2 to. From the left, 7 ÷ 3 = 2 remainder 1. Carry the 1 to make 12, and 12 ÷ 3 = 4. So 72 ÷ 3 = 24.

What is true: Start from the left. What is left over in one place is exchanged for ten of the next place. So you carry it to the right.

KEY WORDS

short division
dividing by a 1 digit number in the bus stop layout.

divisor
the number you divide by.

remainder
what is left over when a number does not divide exactly.

BACK TO YOUR GUESS

The answer is C: Start from the left. Say how many 7s go into each number, and carry what is left over. The answer is 745 with 2 left over.

Division starts from the left, with the biggest place. A leftover is worth ten of the next place. So you carry it on. 5 217 ÷ 7 = 745 with 2 left over.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Work out 3 648 ÷ 4.

   Answer at the back of the book.

2. Work out 4 523 ÷ 6. Give the answer and the remainder.

   Answer at the back of the book.

3. What is 8 125 ÷ 5?

   A 1 001

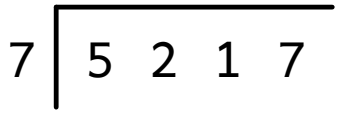
   B 1 625

   C 1 600

   D 1 605

   Answer at the back of the book.

4. In 5 217 ÷ 7, the first digit is 5, which is smaller than 7. What do you do?



A Throw the 5 away and start with the 2.

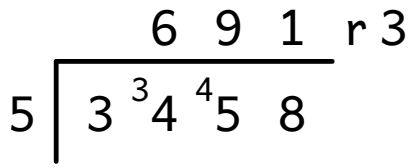
B Write 5 above the bracket and move on.

C Divide 7 by 5 instead.

D Carry it to the next digit. Then divide 52 by 7.

Answer at the back of the book.

5. Read the bus stop for 3 458 ÷ 5. What is the remainder?



A 3

B 8

C 691

D 0

Answer at the back of the book.

6. What is 1 000 ÷ 7? Write the remainder over the divisor, as a fraction.

A $142\ ^{1}/_{7}$

B $142\ ^{7}/_{6}$

C $142\ ^{6}/_{7}$

D $142\ ^{6}/_{10}$

Answer at the back of the book.

7. A rope is 1 500 cm long. Ravi cuts it into pieces of 7 cm. How many full pieces does he get?

   A 215

   B 214

   C 2

   D 214 remainder 2

   Answer at the back of the book.

8. Work out 2 000 ÷ 7. Write the remainder over the divisor, as a fraction. Do not simplify it.

   Answer at the back of the book.

9. **From an earlier chapter**

   Work out 136 × 25.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Divisor outside, number inside. Start at the left. See how many times the divisor goes into each number. Write it above, and carry what is left over. The last leftover is the remainder.

Draw: Draw the bus stop for 5 217 ÷ 7. Write 745 above it, the small carried digits, and r 2 at the end.

M3.2.5 CHAPTER 5

Long division

How do I divide 7 870 by 23 on paper?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Divide each digit by 23, from the left, and write the answers in a row.
- ☐ **B** Divide 7 870 by 2, and then divide the answer by 3.
- ☐ **C** Make a list of the 23 times table. Then find how many 23s fit into each part, and write the working down. The answer is 342 with 4 left over.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.7. If a remainder is a new word, read this first.
- Level 2, Adding and taking away big numbers, M2.2.4. If taking away 69 from 78 in columns is not firm yet, read this first.

BY THE END YOU CAN

- Divide a 4 digit number by a 2 digit number
- Make a list of multiples to help with long division

Mei has 7 870 seeds. She plants them in rows of 23. How many full rows does she make? How many seeds are left over?

Short division is hard when the divisor is big, because the calculations in your head grow. **Long division** writes the working down. First, make a list of **multiples** of 23. A multiple is a number in a times table.

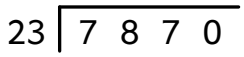
times	answer	times	answer
1	23	6	138
2	46	7	161
3	69	8	184
4	92	9	207
5	115	10	230

The first ten multiples of 23. Each one is 23 more than the one before.

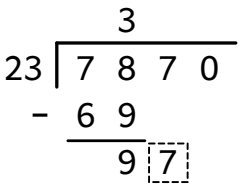
You can make the list by adding 23 each time. Check the last line. 10 × 23 is 23 with a 0 after it, which is 230.

DRAW IT FIRST

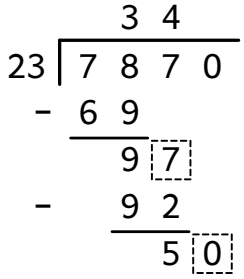
Long division, round by round



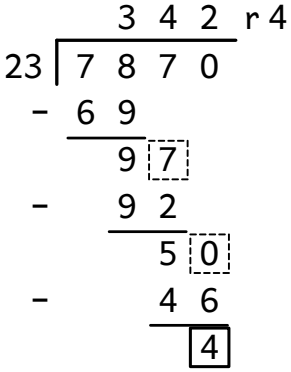
1 Write 23 outside the bracket and 7 870 inside it. 7 is less than 23, so start with 78.



2 3 × 23 = 69, and 4 × 23 is too big. Write 3 above the 8. Take away: 78 − 69 = 9. Bring down the 7 to make 97.



3 4 × 23 = 92. Write 4 above the 7. Take away: 97 − 92 = 5. Bring down the 0 to make 50.



4 2 × 23 = 46. Write 2 above the 0. Take away: 50 − 46 = 4. Nothing is left to bring down, so the answer is 342 remainder 4.

Every round has the same four moves. Divide: how many 23s fit? Multiply: write the product. Take away: find the leftover. Bring down: add the next digit.

WORKED EXAMPLE

Work out 5 376 ÷ 21.

1. Start with 53, because 5 is less than 21. Then 2 × 21 = 42, and 53 − 42 = 11.
2. Bring down the 7 to make 117. Then 5 × 21 = 105, and 117 − 105 = 12.
3. Bring down the 6 to make 126. Then 6 × 21 = 126, and 126 − 126 = 0.
4. Nothing is left over. The answer is 256.

Answer: 256

WORKED EXAMPLE

Work out 9 126 ÷ 34.

1. Start with 91. Then 2 × 34 = 68, and 91 − 68 = 23.
2. Bring down the 2 to make 232. Then 6 × 34 = 204, and 232 − 204 = 28.
3. Bring down the 6 to make 286. Then 8 × 34 = 272, and 286 − 272 = 14.
4. The answer is 268 remainder 14.

Answer: 268 remainder 14

WORKED EXAMPLE

Work out 6 150 ÷ 15.

1. Start with 61. Then 4 × 15 = 60, and 61 − 60 = 1.
2. Bring down the 5 to make 15. Then 1 × 15 = 15, and 15 − 15 = 0.
3. Bring down the 0. Nothing was left over, so the number is 0. 15 goes into 0 no times, so write 0 above it.
4. The answer is 410.

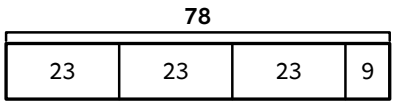
Answer: 410

SEE IT THREE WAYS

With things

Take 78 stones. Make piles of 23 stones. Keep making piles until you cannot make another full pile.

As a picture



In symbols

78 ÷ 23 = 3 remainder 9, because 3 × 23 = 69 and 78 − 69 = 9.

TRY IT

Make your own list of multiples

You need: paper and a pencil.

1. Write 1 × 17 = 17 at the top of the page.
2. Add 17 to get the next line, 2 × 17 = 34. Keep adding 17 down to 10 × 17.
3. Check your last line. It should be 17 with a 0 after it.

What you will see: Each line is the line above plus 17. The fifth line, 85, is half of the last line, 170. A list like this takes one minute and saves many mistakes.

If you cannot try it, this is what happens.

times	answer	times	answer
1	17	6	102
2	34	7	119
3	51	8	136
4	68	9	153
5	85	10	170

A WRONG IDEA MANY PEOPLE HAVE

~~To divide by 23, divide by 2 and then by 3.~~

Why it feels right: Times 6 splits into times 2 and times 3. The digits 2 and 3 sit inside 23.

The test that breaks it: Try 7 870 ÷ 2 = 3 935. Then 3 935 ÷ 3 = 1311 remainder 2. But 7 870 ÷ 23 = 342 remainder 4.

What is true: Dividing by 2 and then by 3 is the same as dividing by 6, because 2 × 3 = 6. The digits of 23 are not two separate divisors.

KEY WORDS

long division
dividing on paper by writing each round of working down.

multiples
the numbers in a times table, such as 23, 46 and 69.

BACK TO YOUR GUESS

The answer is C: Make a list of the 23 times table. Then find how many 23s fit into each part, and write the working down. The answer is 342 with 4 left over.

Long division writes down each round of working, with a list of multiples to help. 7 870 ÷ 23 = 342 with 4 left over. Dividing by 2 and then by 3 is the same as dividing by 6, not by 23.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. In 7 870 ÷ 23, which number do you start with?

$$23 \overline{)\,7\ 8\ 7\ 0}$$

A 78

B 7

C 787

D 23

Answer at the back of the book.

2. The list of multiples of 23 is 23, 46, 69, 92 and 115. How many 23s fit into 97?

times	answer
1	23
2	46
3	69
4	92
5	115

times	answer
6	138
7	161
8	184
9	207
10	230

A 5

B 4

C 3

D 92

Answer at the back of the book.

3. You have worked out 78 − 69 = 9 in 7 870 ÷ 23. What do you do next?

A Bring down the 0 instead of the 7.

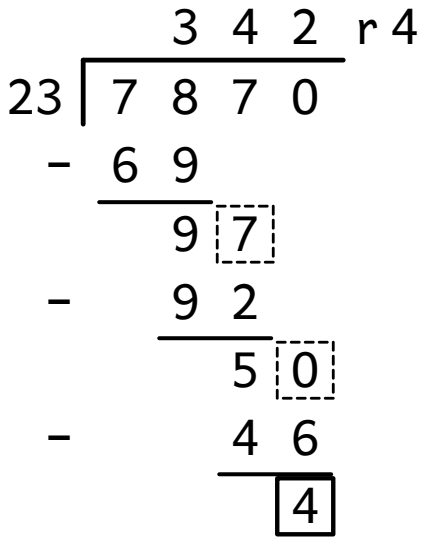
B Write 9 as part of the answer.

C Stop, because 9 is smaller than 23.

D Bring down the 7 next to the 9 to make 97.

Answer at the back of the book.

4. Read the working. What is 7 870 ÷ 23?



A 342 remainder 46

B 342

C 342 remainder 4

D 34 remainder 4

Answer at the back of the book.

5. Which list shows the first five multiples of 17?

A 17, 34, 51, 68, 85

B 17, 27, 37, 47, 57

C 17, 34, 68, 136, 272

D 34, 51, 68, 85, 102

Answer at the back of the book.

6. Work out 4 836 ÷ 12.

Answer at the back of the book.

7. Work out 9 126 ÷ 34. Give the answer and the remainder.

Answer at the back of the book.

8. Write the first five multiples of 17.

   Answer at the back of the book.

9. **From an earlier chapter**

   Work out 6 207 ÷ 9. Give the answer and the remainder.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Make a list of multiples of the divisor. Then repeat four moves: divide, multiply, take away, bring down. The last leftover is the remainder.

Draw: Draw the long division of 7 870 by 23. Write the list of multiples of 23 beside it.

M3.2.6 CHAPTER 6

Order of operations

What is 3 + 4 × 2?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 14, because you work from left to right: 3 + 4 = 7, then 7 × 2 = 14.
- ☐ **B** 11, because multiplying comes before adding: 4 × 2 = 8, then 3 + 8 = 11.
- ☐ **C** 9, because you add all the numbers: 3 + 4 + 2 = 9.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.1. If 4 × 2 as groups is new to you, read this first.
- Level 2, Times tables and sharing, M2.3.6. If you are not sure how times and divide go together, read this first.

BY THE END YOU CAN

- Work out 3 + 4 × 2 in the right order
- Use brackets to change the order of working
- Put in brackets to make 2 + 3 × 4 equal 20

Ana has 3 loose beans and 4 bags. Each bag holds 2 beans. She writes the calculation 3 + 4 × 2. How many beans does she have?

A calculation with more than one operation must give one answer, for everybody. So we agree on an **order of operations**. First, count the beans in the picture.

DRAW IT FIRST

Count the beans, then write the calculation



1 Ana has 3 loose beans.

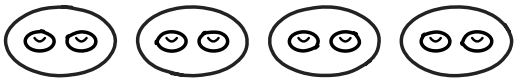




4 bags of 2

2 She also has 4 bags with 2 beans in each. That is 4 × 2 = 8 beans in the bags.





4 bags of 2

3 + 4 × 2 = 3 + 8 = 11

3 Add the loose beans and the bags. 3 + 8 = 11. So 3 + 4 × 2 = 11.

The beans match the agreed order. The bags are counted as 4 × 2 first, and the loose beans are added after. So multiplying and dividing come before adding and taking away.

first	next	last
brackets	× and ÷	+ and −

The order of working. Inside each box, work from left to right.

Brackets come first of all. Work out what is inside them before anything else. Multiplying and dividing share one step. Adding and taking away share the next. Inside a step, work from left to right.

WORKED EXAMPLE

Work out 3 + 4 × 2.

1. There are no brackets.
2. Multiply first. 4 × 2 = 8.
3. Now the calculation is 3 + 8. Add. 3 + 8 = 11.

Answer: 11

WORKED EXAMPLE

Work out 20 − 12 ÷ 4 + 2.

1. There are no brackets. Divide first. 12 ÷ 4 = 3.
2. Now the calculation is 20 − 3 + 2. Subtracting and adding share a step, so work from left to right.
3. 20 − 3 = 17.
4. 17 + 2 = 19.

Answer: 19

SEE IT THREE WAYS

With things	**As a picture**	**In symbols**
Put the 3 white beans and 4 black beans in one bag. Make 2 bags like that.	 bag 1 bag 2	**(3 + 4) × 2 = 7 × 2 = 14. The brackets say: add first.**

Brackets change the order. 3 + 4 × 2 is 11, but (3 + 4) × 2 is 14. In the second calculation the adding is inside the brackets, so it happens first.

WORKED EXAMPLE

Work out (8 − 2) × 3 + 12 ÷ 4.

1. Brackets first. 8 − 2 = 6.
2. The calculation is now 6 × 3 + 12 ÷ 4. Multiply and divide next.
3. 6 × 3 = 18.
4. 12 ÷ 4 = 3.
5. Add. 18 + 3 = 21.

Answer: 21

TRY IT

Make 14 and make 20 from the same cards

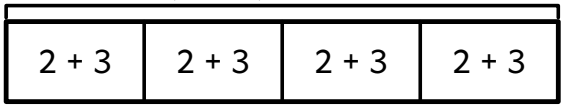
You need: 3 small pieces of paper with 2, 3 and 4 written on them, a pencil.

1. Write the calculation 2 + 3 × 4. Work it out in the right order. Use 3 bags of 4 beans and 2 loose beans if it helps.
2. Now put brackets round 2 + 3, to make (2 + 3) × 4. Work it out.
3. Compare the two answers.

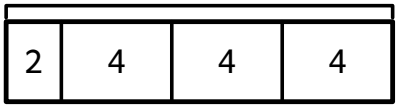
What you will see: The cards are the same, but the answers are different. 2 + 3 × 4 = 14. (2 + 3) × 4 = 20. The brackets changed the order.

If you cannot try it, this is what happens.

(2 + 3) × 4 = 20



2 + 3 × 4 = 14



A WRONG IDEA MANY PEOPLE HAVE

~~Always work from left to right.~~

Why it feels right: We read words from left to right, and the first signs you learned, + and −, are worked that way.

The test that breaks it: Try 3 + 4 × 2 from left to right. 3 + 4 = 7 and 7 × 2 = 14. But Ana's beans make 11, and the agreed order gives 11 too. Left to right gives 14, which does not match the beans.

What is true: Brackets first. Then multiply and divide. Then add and take away. Left to right works only inside a step.

KEY WORDS

order of operations
the agreed order for working out a calculation: brackets, then × and ÷, then + and −.

brackets
the marks ( ) round a part of a calculation that is worked out first.

BACK TO YOUR GUESS

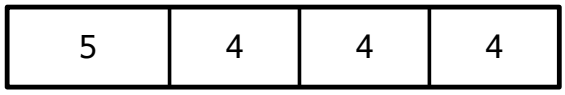
The answer is B: 11, because multiplying comes before adding: 4 × 2 = 8, then 3 + 8 = 11.

The answer is 11. Multiplying comes before adding, so 4 × 2 = 8 first, and then 3 + 8 = 11. Everyone uses the same order, so every calculation has just one answer.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. What is 5 + 3 × 4?



A 60

B 32

C 12

D 17

Answer at the back of the book.

2. What is 18 ÷ 3 × 2?

A 12

B 3

C 108

Answer at the back of the book.

3. What is (6 + 2) × 5?

A 13

B 16

C 40

Answer at the back of the book.

4. Where do the brackets go to make 2 + 3 × 4 equal 20?

A 2 + (3 × 4) = 14

B (2 + 3) × 4 = 20

C (2 + 3 × 4) = 14

Answer at the back of the book.

5. The picture shows 2 bags. Each bag holds 3 white beans and 4 black beans. Which calculation gives all the beans?



bag 1 bag 2

A $(3 + 4) \times 2 = 14$

B $3 + 4 \times 2 = 11$

C $3 \times 2 + 4 = 10$

Answer at the back of the book.

6. Work out $8 + 6 \times 3$.

Answer at the back of the book.

7. Work out $(7 + 5) \times 4$.

Answer at the back of the book.

8. Work out $30 - 12 \div 3 + 4$.

Answer at the back of the book.

9. **From an earlier chapter**

Work out $5\,928 \div 24$.

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Order: brackets first. Then × and ÷. Then + and −. Inside a step, work left to right. 3 + 4 × 2 = 11, but (3 + 4) × 2 = 14.

Draw: Draw 3 loose beans and 4 bags of 2 beans. Write 3 + 4 × 2 = 11 under it.

M3.2.7 CHAPTER 7

Estimate, then check

How can I tell if 389 × 52 = 20 228 is about right?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** If the answer has many digits, it must be right.
- ☐ **B** Add the numbers: 389 + 52 = 441. The answer should be near 441.
- ☐ **C** Round each number to one digit: 400 × 50 = 20 000. The answer is close, so it is about right.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Big numbers and place value, M2.1.5. If rounding to the nearest 10 or 100 is new to you, read this first.
- Level 2, Adding and taking away big numbers, M2.2.6. If estimating before you work out is new to you, read this first.

BY THE END YOU CAN

- Round to one digit to estimate 389 × 52
- Check a division by multiplying back

Kofi works out 389 × 52 on paper and gets 20 228. Is that about right? One slip with a carry could be a long way out. A quick **estimate** will tell him.

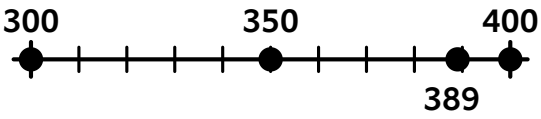
An estimate is a rough answer, worked out in your head. To make one, **round** each number to one digit. Keep the first digit, and fill the rest with zeros. If the next digit is 5 or more, round up.

DRAW IT FIRST

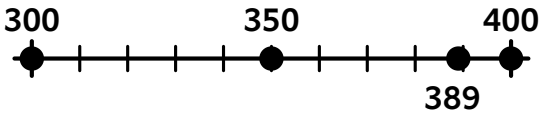
Round 389 to one digit



1 389 is between 300 and 400. Half way between them is 350. Mark 300, 350 and 400.



2 Mark 389. It is past 350.



389 rounds to 400

3 389 is nearer to 400 than to 300, so it rounds to 400.

Do the same with 52. It is between 50 and 60, and the half way mark is 55. 52 is before 55, so it rounds down to 50. Then 389 × 52 is about 400 × 50.

	400
50	20 000

total 20 000

not to scale

The estimate, as a grid with one box.

WORKED EXAMPLE

Estimate 389 × 52.

1. Round each number to one digit. 389 is about 400. 52 is about 50.
2. Work out 4 × 5 = 20. There are two zeros in 400 and one in 50, so put back three zeros.
3. 400 × 50 = 20 000.
4. So 389 × 52 is about 20 000.

Answer: 20 000

SEE IT THREE WAYS

With things

A farmer plants 52 rows of trees, with 389 trees in each row. Say the numbers roughly: about 50 rows, and about 400 trees in each row.

As a picture

number	rounded
389	400
52	50

In symbols

389 × 52 is about
400 × 50 = 20 000.

An estimate catches big slips, such as a lost zero. It cannot catch a small slip. Multiplying back catches those. Division and multiplying are opposites. If 7 870 ÷ 23 = 342 remainder 4, then 342 × 23 + 4 must be 7 870.

WORKED EXAMPLE

Check that 7 870 ÷ 23 = 342 remainder 4 by multiplying back.

1. Multiply the answer by the divisor. Work out 342 × 23.
2. Row 1: 342 × 3 = 1 026.
3. Row 2: 342 × 20 = 6 840.
4. Add the rows. 1 026 + 6 840 = 7 866.
5. Add the remainder. 7 866 + 4 = 7 870. This is the number we started with, so the division is right.

Answer: 7 870

WORKED EXAMPLE

Estimate 7 870 ÷ 23. Is 342 about right?

1. Round 7 870 to one digit. It is 8 000.
2. Round 23 to one digit. It is 20.
3. Cross out one zero from each number. 8 000 ÷ 20 is the same as 800 ÷ 2.
4. Divide. 800 ÷ 2 = 400.
5. The estimate is 400. Both numbers were rounded so that the estimate grew: 7 870 went up and 23 went down. So the true answer should be a little under 400. 342 is in the hundreds, like 400, so no digit is in the wrong place.

Answer: 400

TRY IT

Multiply back

You need: paper and a pencil.

1. Work out 7 870 ÷ 23 on paper. You should get 342 remainder 4.
2. Now multiply your answer by the divisor. Work out 342 × 23 in columns.
3. Add the remainder to what you get.

What you will see: 342 × 23 = 7 866. Adding the 4 left over gives 7 870, the number you began with. If it does not match, one of the two calculations has a slip.

If you cannot try it, this is what happens.

$$\begin{array}{r} 342 \\ \times\ \ 23 \\ \hline 1026 \\ +\ 6840 \\ \hline 7866 \end{array}$$

A WRONG IDEA MANY PEOPLE HAVE

~~To check a division, just divide again.~~

Why it feels right: Doing the same calculation twice feels like a fair test.

The test that breaks it: If you made a slip the first time, you may make the same slip again. Multiplying back is a different move. 342 × 23 + 4 = 7 870 proves the first answer.

What is true: Check with the opposite operation. Multiply the answer by the divisor, add the remainder, and you must get the number you began with.

KEY WORDS

estimate
a rough *a*nswer, worked out quickly from rounded numbers.

round
to ch*a*nge *a* number to *a* nearby number th*a*t is e*a*sier to use, such *a*s 389 to 400.

BACK TO YOUR GUESS

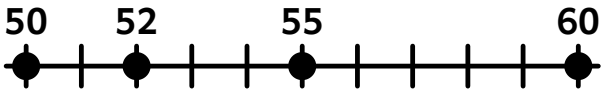
The *a*nswer is C: Round e*a*ch number to one digit: 400 × 50 = 20 000. The *a*nswer is close, so it is *a*bout right.

Rounding to one digit gives 400 × 50 = 20 000. The *a*nswer 20 228 is close to th*a*t, so it is *a*bout right. Adding is the wrong oper*a*tion, *a*nd the length of *a*n *a*nswer proves nothing.

CHECK YOURSELF

Try e*a*ch one before you look. A wrong *a*nswer tells you where to go next.

1. Round 52 to one digit. Which number is it?



A 52

B 60

C 55

D 50

Answer *a*t the b*a*ck of the book.

2. Which estimate is best for 389 × 52?

   A 300 × 50 = 15 000

   B 400 × 50 = 20 000

   C 400 × 5 = 2 000

   D 400 + 50 = 450

   Answer at the back of the book.

3. Ana works out 1 000 ÷ 7 and gets 142 remainder 6. How can she check?

   A Multiply 142 by 7 and ignore the 6.

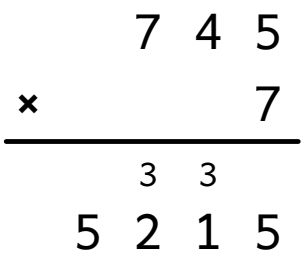
   B Divide 142 by 7.

   C Add 142 and 7.

   D Multiply 142 by 7, then add 6. The result should be 1 000.

   Answer at the back of the book.

4. The picture shows 745 × 7 = 5 215. Ravi divided 5 217 by 7 and got 745 remainder 2. Is his division right?



   A Yes, because 5 215 + 2 = 5 217.

   B No, because 5 215 is not 5 217.

   C No, because the remainder should be 0.

   D Yes, because 745 × 7 is close to 5 217.

   Answer at the back of the book.

5. Which is a good estimate for 7 870 ÷ 23?

   A 8 000 ÷ 2 = 4 000

   B 800 ÷ 20 = 40

   C 8 000 ÷ 20 = 400

   Answer at the back of the book.

6. Round each number to one digit to estimate 612 × 38. Give the estimate.

   Answer at the back of the book.

7. Dividing by 7 gave 814 remainder 3. Multiply back to find the number that was divided.

   Answer at the back of the book.

8. **From an earlier chapter**

   Work out 9 + 4 × 6.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Estimate: round each number to one digit, then work it out. 389 × 52 is about 400 × 50 = 20 000. Check a division by multiplying back, then add the remainder.

Draw: Draw a number line from 300 to 400. Mark 350 in the middle and 389 near the end. Write "rounds to 400".

M3.2.8 CHAPTER 8

Choosing what to do

How do I know whether to multiply or divide?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Look for the word "each". It always means multiply.
- ☐ **B** Draw a bar model. With equal parts, multiply to find the total and divide to find one part. With unequal parts, add and take away.
- ☐ **C** Always add first, then multiply, then divide.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.3. If you are not sure when to share and when to group, read this first.
- Level 2, Adding and taking away big numbers, M2.2.7. If a bar model for adding is new to you, read this first.

BY THE END YOU CAN

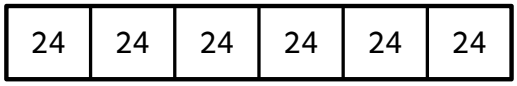
- Solve a problem with several steps using all four operations
- Draw a bar model to decide whether to multiply or divide

Joseph reads: "6 boxes hold 24 pencils each. How many pencils are there?" Should he multiply or divide? Many people guess. A drawing can decide for you.

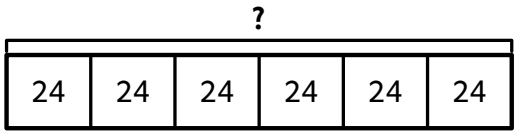
A **bar model** is a bar cut into parts. The whole bar is the total. Write what you know on the parts. Write a ? where the answer goes.

DRAW IT FIRST

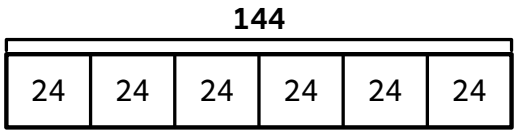
A bar model for 6 boxes of 24 pencils



1 Draw 6 equal parts for the 6 boxes. Write 24 in each part, because each box holds 24 pencils.

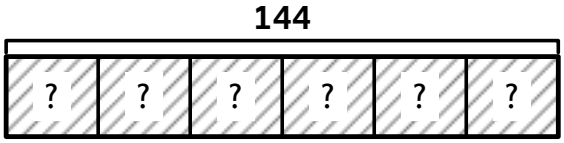


2 The whole bar is the total number of pencils. We do not know it yet, so write a question mark over the bar.



3 The parts are equal and we want the total, so multiply. 6 × 24 = 144 pencils.

Now turn the problem round. 144 pencils are shared between 6 boxes. How many go in each box? The total is known. The parts are what we want.



The total is known and each part is wanted, so divide: 144 ÷ 6 = 24.

Here is the rule. Equal parts: from the parts to the total, multiply. From the total to one part, or to the number of parts, divide. Parts that are not equal: add them for the total. Take away to find a missing part. The bar shows which one you have.

WORKED EXAMPLE

Mei has 3 bags with 24 beads in each, and 18 loose beads. How many beads does she have?

1. Draw a bar with 3 parts of 24 and one part of 18. The whole bar is the total, so write a question mark over it.
2. The bags: 3 × 24 = 72.
3. Add the loose beads. 72 + 18 = 90.

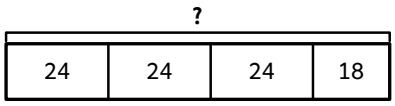
Answer: 90

SEE IT THREE WAYS

With things

Lay out 3 bags of 24 beads in a row, and then 18 loose beads at the end.

As a picture



In symbols

3 × 24 + 18 = 72 + 18 = 90.

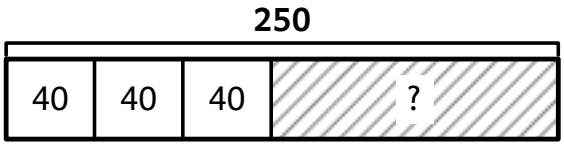
WORKED EXAMPLE

Mei shares her 90 beads equally between 6 friends. How many beads does each friend get?

1. The total is known, and so is the number of parts. We want the size of one part, so divide.
2. Use short division. 6 goes into 9 once, leaving 3. 6 goes into 30 five times.
3. 90 ÷ 6 = 15.
4. Check by multiplying back. 6 × 15 = 90.

Answer: 15

Many problems have several steps. Draw the bar for the first step, then the next. Each step uses one **operation**, which is adding, taking away, multiplying or dividing. You may need all four.



Hana gives 40 to each of 3 friends. The part with the question mark is what is left.

WORKED EXAMPLE

Hana has 250 stickers. She gives 40 to each of 3 friends. She shares the rest equally between 5 cousins. Each cousin then finds 4 more stickers. How many stickers does each cousin have now?

1. Draw a bar for the 250 stickers. Three parts of 40 are given away. The last part is what is left.
2. Stickers given away: 3 × 40 = 120.
3. Stickers left: 250 − 120 = 130.
4. Each cousin gets 130 ÷ 5 = 26.
5. Each cousin finds 4 more. 26 + 4 = 30.

Answer: 30

TRY IT

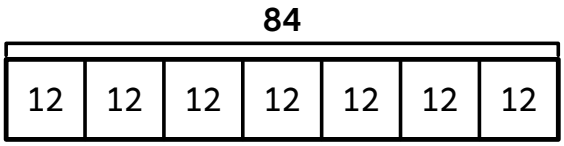
Draw a bar for 84 eggs

You need: paper and a pencil, a ruler, if you have one.

1. Draw a long bar and write 84 over it. A farmer packs 84 eggs into trays of 12.
2. Mark off parts of 12 along the bar, one after another, until the bar is full.
3. Count the parts. That is the number of trays.

What you will see: You make 7 parts of 12. So the farmer fills 7 trays. The total is known and the number of parts is wanted, so the calculation is 84 ÷ 12 = 7.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The word "each" always means multiply.~~

Why it feels right: In "4 boxes with 6 pencils in each", the word each goes with multiplying.

The test that breaks it: Try "12 pencils are shared between 3 boxes. How many go in each box?" The word each is there, but the calculation is 12 ÷ 3 = 4.

What is true: Do not hunt for words. Draw the bar. With equal parts, parts to total means multiply, and total to a part means divide. With unequal parts, add or take away.

KEY WORDS

bar model
a bar cut into parts that shows what you know and what you want to find.

operation
one of adding, taking away, multiplying and dividing.

BACK TO YOUR GUESS

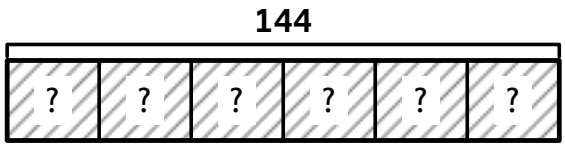
The answer is B: Draw a bar model. With equal parts, multiply to find the total and divide to find one part. With unequal parts, add and take away.

A drawing shows what you know and what you want. Equal parts: from the parts to the total, multiply. From the total to one part, divide. Unequal parts: add them for the total, and take away to find a missing part. The word "each" can go with either one.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. The bar shows 144 pencils shared in 6 equal parts. Which calculation finds one part?



A 144 + 6 = 150

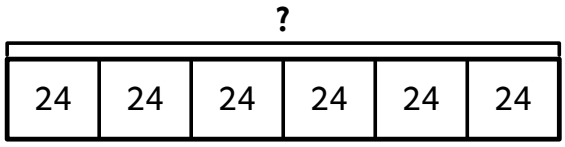
B 144 × 6 = 864

C 144 − 6 = 138

D 144 ÷ 6 = 24

Answer at the back of the book.

2. The bar shows 6 equal parts of 24 pencils each. Which calculation finds the total?



A 24 + 6 = 30

B 24 ÷ 6 = 4

C 6 × 24 = 144

Answer at the back of the book.

3. Mei has 3 bags of 24 beads and 18 loose beads. She shares all the beads equally between 6 friends. How many beads does each friend get?

A 15

B 12

C 75

D 540

Answer at the back of the book.

4. A rope is 156 m long. Ravi cuts it into pieces of 12 m. How many pieces does he get?

A 1 872

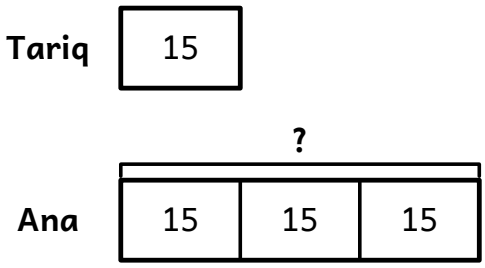
B 13

C 144

D 168

Answer at the back of the book.

5. Ana has 3 times as many stickers as Tariq. Tariq has 15 stickers. The bars show this. How many stickers does Ana have?



A 12

B 5

C 18

D 45

Answer at the back of the book.

6. Mei has 4 bags of 25 beads and 36 loose beads. She shares all the beads equally between 8 friends. How many beads does each friend get?

   Answer at the back of the book.

7. Hana has 250 stickers. She gives 40 to each of 3 friends. She shares the rest equally between 5 cousins. How many stickers does each cousin get?

   Answer at the back of the book.

8. **From an earlier chapter**

   Round each number to one digit to estimate 487 × 31. Give the estimate.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Draw a bar model. Equal parts: multiply for the total, divide for one part. Unequal parts: add, or take away a part. Do one operation at a time, and check the answer makes sense.

Draw: Draw 6 equal parts with 24 in each, and a bracket over them. Write 6 × 24 = 144.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

bar model
a bar cut into parts that shows what you know and what you want to find. M3.2.8

brackets
the marks ( ) round a part of a calculation that is worked out first. M3.2.6

carry
to move the tens of a product to the next place on the left. M3.2.2

divisor
the number you divide by. M3.2.4

doubling
making a number twice as big. M3.2.1

estimate
a rough answer, worked out quickly from rounded numbers. M3.2.7

grid method
splitting a number by place value and multiplying each part in its own box. M3.2.2

halving
cutting a number into two equal parts. M3.2.1

long division
dividing on paper by writing each round of working down. M3.2.5

multiples
the numbers in a times table, such as 23, 46 and 69. M3.2.5

multiples of ten
numbers you get by multiplying a whole number by 10, such as 30, 40 and 700. M3.2.1

operation
one of adding, taking away, multiplying and dividing. M3.2.8

order of operations
the agreed order for working out a calculation: brackets, then × and ÷, then + and −. M3.2.6

partial product
one row of a long multiplication, a part of the full answer. M3.2.3

placeholder
a 0 that holds a place empty, so the other digits stay in the right places. M3.2.3

remainder
what is left over when a number does not divide exactly. M3.2.4

round
to change a number to a nearby number that is easier to use, such as 389 to 400. M3.2.7

short division
dividing by a 1 digit number in the bus stop layout. M3.2.4

Answers

M3.2.1. Multiplying and dividing in your head

1. **6 700**

   Multiplying by 100 moves each digit two places to the left. 67 becomes 6 700.

2. **80**

   Dividing by 100 moves each digit two places to the right. 8 000 becomes 80.

3. **50 × 8**

   Double 25 to make 50 and halve 16 to make 8. 50 × 8 = 400, the same as 25 × 16.

   - **If you chose B:** You doubled both numbers. That makes the answer four times bigger. 50 × 32 = 1 600, not 400. Go to M3.2.1.
   - **If you chose C:** You halved 16 but did not double 25. That halves the answer. 25 × 8 = 200, not 400. Go to M3.2.1.
   - **If you chose D:** 25 is odd, so it does not halve into a whole number. 12 × 32 = 384, not 400. Go to M3.2.1.

4. **Three places to the left, so the answer is 62 000.**

   1 000 has three zeros, so each digit moves three places to the left. 62 becomes 62 000.

   - **If you chose A:** Multiplying is not adding. Multiplying by 1 000 makes each digit worth 1 000 times as much. Go to M3.2.1.
   - **If you chose B:** That is multiplying by 10. Each zero in 1 000 moves the digits one more place, and 1 000 has three zeros. Go to M3.2.1.
   - **If you chose D:** Moving to the right makes a number smaller. That is dividing. Multiplying moves the digits to the left. Go to M3.2.1.

5. **450**

   Dividing by 10 moves each digit one place to the right. 4 500 becomes 450.

   - **If you chose A:** Dividing by 10 always changes the number. Each digit moves one place to the right. Go to M3.2.1.
   - **If you chose B:** That is dividing by 100. Dividing by 10 moves the digits only one place to the right. Go to M3.2.1.
   - **If you chose C:** The digits moved to the left. That is multiplying by 10. Dividing moves them to the right. Go to M3.2.1.

6. **420**

   70 × 6 = 420. This is the same as 35 × 12, because doubling one number and halving the other keeps the answer.

   - **If you chose A:** You may have doubled both numbers, so 70 × 12. Halve 12 when you double 35. 70 × 6 uses the halved number. Go to M3.2.1.
   - **If you chose C:** You may have used 35 × 6. Both changes are needed: double 35 and halve 12, then multiply 70 by 6. Go to M3.2.1.
   - **If you chose D:** You added 70 and 6. The calculation is a multiplication, so work out 70 × 6. Go to M3.2.1.

7. **3 000**

   Work out 5 × 6 = 30, then put back two zeros. 50 × 60 = 3 000.

   - **If you chose B:** You worked out 5 × 6 = 30, then put back only one zero. Each number has a zero, so put back two. Go to M3.2.1.
   - **If you chose C:** You put back three zeros. 50 and 60 have one zero each, so two zeros go back after 30. Go to M3.2.1.
   - **If you chose D:** You added 50 and 60. The box holds the product, so multiply. Go to M3.2.1.

8. **4 200**

   First work out 6 × 7 = 42. Then put back the two zeros. 60 × 70 = 4 200.

9. **490**

   Double 35 to make 70. Halve 14 to make 7. Then 70 × 7 = 490.

M3.2.2. Short multiplication

1. **735**

   Split 245 into 200, 40 and 5. 3 × 200 = 600, 3 × 40 = 120 and 3 × 5 = 15. 600 + 120 + 15 = 735.

2. **92**

   4 × 23 is 8 tens and 12 ones. Trade 10 ones for 1 ten. That gives 9 tens and 2 ones, which is 92.

3. **1 254**

   Split 418 into 400, 10 and 8. 3 × 400 = 1 200, 3 × 10 = 30 and 3 × 8 = 24. 1 200 + 30 + 24 = 1 254.

   - **If you chose A:** You may have written each product side by side: 12, 3 and 24. The digits are worth different amounts, so carry the tens. Go to M3.2.2.
   - **If you chose B:** You may have dropped the carried 2. 3 × 8 = 24, so 2 is carried. 3 × 1 = 3, and 3 + 2 = 5, not 3. Go to M3.2.2.
   - **If you chose C:** You may have added the carried 2 to the 1 first and then multiplied. Multiply first, then add the carry. Go to M3.2.2.

4. **120**

   The 4 in 245 is worth 40. 3 × 40 = 120.

   - **If you chose B:** You used the digit 4. But it is 4 tens, so its value is 40. 3 × 40 = 120. Go to M3.2.2.
   - **If you chose C:** You added 3 and 40. The grid multiplies each part by 3. Go to M3.2.2.
   - **If you chose D:** That is 3 × 200, the box under the 200. The box under the 40 is 3 × 40. Go to M3.2.2.

5. **1 200, 80 and 24**

   Split 326 into 300, 20 and 6. 4 × 300 = 1 200, 4 × 20 = 80 and 4 × 6 = 24.

   - **If you chose A:** These are 4 × 3, 4 × 2 and 4 × 6. The 3 is 300 and the 2 is 20, so the boxes are worth more. Go to M3.2.2.
   - **If you chose C:** These are the parts of 326 before they are multiplied. Each part must be multiplied by 4. Go to M3.2.2.
   - **If you chose D:** The 6 must be multiplied by 4 too. 4 × 6 = 24. Go to M3.2.2.

6. **Write 8 and carry 4.**

   48 is 4 tens and 8 ones. You write the 8 in the ones place and carry the 4 tens to the tens place.

   - **If you chose A:** The 4 is 4 tens. It is worth 40, so it must be added to the next product. Go to M3.2.2.
   - **If you chose B:** Each place holds one digit. Write the 8 and carry the 4 tens to the next place. Go to M3.2.2.
   - **If you chose C:** The digit you write is the ones digit, 8. The 4 is 4 tens, and it is carried. Go to M3.2.2.

7. **6 092**

   Ones: 3 × 4 = 12, write 2 and carry 1. Tens: 2 × 4 = 8 and 8 + 1 = 9. Hundreds: 5 × 4 = 20, write 0 and carry 2. Thousands: 1 × 4 = 4 and 4 + 2 = 6.

   - **If you chose A:** You may have dropped the carried 1 from 3 × 4 = 12. Then 2 × 4 + 1 = 9, not 8. Go to M3.2.2.
   - **If you chose B:** You may have dropped the carry of 2 from 5 × 4 = 20. Then 1 × 4 + 2 = 6, not 4. Go to M3.2.2.
   - **If you chose D:** You may have written each product side by side: 4, 20, 8 and 12. Carry the tens instead. Go to M3.2.2.

8. **21 434**

   Split 3 062 into 3 000, 60 and 2. There is no hundreds box. 7 × 3 000 = 21 000, 7 × 60 = 420 and 7 × 2 = 14. 21 000 + 420 + 14 = 21 434.

9. **12 000**

   Work out 4 × 3 = 12. Then put back the three zeros, two from 400 and one from 30. 400 × 30 = 12 000.

M3.2.3. Long multiplication

1. **312**

   Row 1: 24 × 3 = 72. Row 2: 24 × 10 = 240. Add the rows. 72 + 240 = 312.

2. **9 310**

   Row 1: 245 × 8 = 1 960. Row 2: 245 × 30 = 7 350. Add the rows. 1 960 + 7 350 = 9 310.

3. **8 820**

   Row 1: 245 × 6 = 1 470. Row 2: 245 × 30 = 7 350. Add the rows. 1 470 + 7 350 = 8 820.

   - **If you chose A:** This is only row 2, 245 × 30. The 6 ones still need their own row, 245 × 6 = 1 470. Go to M3.2.3.
   - **If you chose B:** You may have left out the 0 in row 2. 245 × 3 = 735, but the 3 is worth 30, so row 2 is 7 350, not 735. Go to M3.2.3.
   - **If you chose C:** This is only row 1, 245 × 6. The 3 tens still need their own row, 245 × 30 = 7 350. Go to M3.2.3.

4. **The digit is worth tens. The row is ten times as big, and the zero holds the ones place.**

   The 5 in 52 means 5 tens. So row 2 is 346 × 50, and the zero holds the ones place.

   - **If you chose B:** The zero does a job. Without it, the digits of row 2 sit one place too far right. Each is worth ten times too little. Go to M3.2.3.
   - **If you chose C:** Not every number times 5 ends in a zero. 347 × 5 = 1 735. The zero is there because the 5 means 5 tens. Go to M3.2.3.
   - **If you chose D:** Row 1 is 346 × 2 = 692, which does not end in a zero. The zero belongs to row 2, the row for the tens. Go to M3.2.3.

5. **645 and 8 600**

   Row 1 is 215 × 3 = 645. Row 2 is 215 × 40 = 8 600.

   - **If you chose A:** Row 2 is 215 × 40, and 215 × 4 = 860 is only the start. The 4 is worth 40, so row 2 is 8 600. Go to M3.2.3.
   - **If you chose C:** You may have started with the 4. Row 1 is for the ones digit, 3. Row 2 is for the 4 tens. Go to M3.2.3.
   - **If you chose D:** You put back one zero too many. The 4 is 4 tens, so row 2 is 215 × 40 = 8 600. Go to M3.2.3.

6. **The second row should be 3 840, not 384. He left out the zero.**

   The 3 in 37 means 3 tens, so row 2 is 128 × 30 = 3 840. Then 896 + 3 840 = 4 736.

   - **If you chose A:** Check with an estimate. 130 × 40 is about 5 200, so 1 280 is far too small. Go to M3.2.3.
   - **If you chose B:** 896 + 384 = 1 280 is right. The slip is in row 2, which needs its zero. Go to M3.2.3.
   - **If you chose C:** Row 1 is for the ones digit, 7, so row 1 is 128 × 7 = 896. That row is right. Go to M3.2.3.

7. **15 000, 2 000 and 300**

   The row for the 50 holds 50 × 300 = 15 000, 50 × 40 = 2 000 and 50 × 6 = 300. They add to 17 300, which is row 2.

   - **If you chose B:** These are the boxes for the 2, the bottom row. They add up to 692, which is row 1. Go to M3.2.3.
   - **If you chose C:** These are the boxes in the first column. Row 2 is the whole row for the 50, across all three columns. Go to M3.2.3.
   - **If you chose D:** These are the pieces of 346, the heads of the columns. The boxes below them hold the products. Go to M3.2.3.

8. **14 736**

   Row 1: 307 × 8 = 2 456. Row 2: 307 × 40 = 12 280. Add the rows. 2 456 + 12 280 = 14 736.

9. **5 052**

   Split 1 263 into 1 000, 200, 60 and 3. The boxes are 4 000, 800, 240 and 12. 4 000 + 800 + 240 + 12 = 5 052.

M3.2.4. Short division

1. **912**

   4 goes into 3 no times. 4 goes into 36 nine times, with nothing left. 4 goes into 4 once. 4 goes into 8 twice. The answer is 912.

2. **753 remainder 5**

   6 goes into 45 seven times, leaving 3. 6 goes into 32 five times, leaving 2. 6 goes into 23 three times, leaving 5. The answer is 753 remainder 5.

3. **1 625**

   5 goes into 8 once, leaving 3. 5 goes into 31 six times, leaving 1. 5 goes into 12 twice, leaving 2. 5 goes into 25 five times. The answer is 1 625.

   - **If you chose A:** You may have divided each digit on its own. The 3 left over from 8 must be carried to make 31, not dropped. Go to M3.2.4.
   - **If you chose C:** This is only 8 000 ÷ 5. The last 125 still needs dividing, and 125 ÷ 5 = 25. Go to M3.2.4.
   - **If you chose D:** You may have dropped the carried 1 after 31 ÷ 5 = 6. Then 2 ÷ 5 gives 0, not 2. Check by multiplying back: 5 × 1 605 = 8 025, not 8 125. Go to M3.2.4.

4. **Carry it to the next digit. Then divide 52 by 7.**

   7 goes into 5 no times, so the 5 is carried to make 52. Then work out 52 ÷ 7.

   - **If you chose A:** The 5 is 5 thousands, so it is worth 5 000. It must be carried on, not thrown away. Go to M3.2.4.
   - **If you chose B:** 7 goes into 5 no times, so 5 is not an answer digit. The 5 is still worth dividing, so carry it. Go to M3.2.4.
   - **If you chose C:** The number inside the bracket is the one that is divided. The number outside is the divisor. Go to M3.2.4.

5. **3**

   5 × 691 = 3 455, and 3 458 − 3 455 = 3. The remainder is 3.

   - **If you chose B:** The last digit of the number is 8, but that is not the remainder. The remainder is what is left after the last step. Go to M3.2.4.
   - **If you chose C:** 691 is the answer to the division, the number in each share. The remainder is the small number after the r. Go to M3.2.4.
   - **If you chose D:** There is a leftover. 5 × 691 = 3 455, and 3 458 is 3 more. Go to M3.2.4.

6. **142 $^{6}/_{7}$**

   7 × 142 = 994, and 1 000 − 994 = 6. The remainder over the divisor is $^{6}/_{7}$.

   - **If you chose A:** The fraction needs the remainder on top, not 1. 7 × 142 = 994, and 1 000 − 994 = 6. Go to M3.2.4.
   - **If you chose B:** The fraction is upside down. The remainder goes on top, and the divisor goes on the bottom. Go to M3.2.4.
   - **If you chose D:** You may have read 142 r 6 as 142.6, which means 6 tenths. The remainder is 6 out of 7 parts, so the fraction is $^{6}/_{7}$. Go to M3.2.4.

7. **214**

   1 500 ÷ 7 = 214 remainder 2. So there are 214 full pieces, and 2 cm of rope is left over.

   - **If you chose A:** The last piece would need 7 cm, but only 2 cm are left. A short piece is not a full piece. Go to M3.2.4.
   - **If you chose C:** The remainder is how many centimetres are left over, not how many pieces are cut. Go to M3.2.4.
   - **If you chose D:** The question asks for full pieces only. The remainder is the leftover rope in centimetres. Go to M3.2.4.

8. **285 $^{5}/_{7}$**

   7 × 285 = 1 995, and 2 000 − 1 995 = 5. The remainder 5 over the divisor 7 is $^{5}/_{7}$.

9. **3 400**

   Row 1: 136 × 5 = 680. Row 2: 136 × 20 = 2 720. Add the rows. 680 + 2 720 = 3 400.

M3.2.5. Long division

1. **78**

   7 is smaller than 23, so 23 cannot go into it. The first two digits make 78, and 23 goes into 78 three times.

   - **If you chose B:** 23 does not go into 7, because 7 is smaller. Take one more digit, so the number is 78. Go to M3.2.5.
   - **If you chose C:** That is one digit too many. 23 already goes into 78, so start there. Go to M3.2.5.
   - **If you chose D:** 23 is the divisor, the number you divide by. You start with digits from the number inside the bracket. Go to M3.2.5.

2. **4**

   4 × 23 = 92 is not more than 97, and 5 × 23 = 115 is. So four 23s fit.

   - **If you chose A:** 5 × 23 = 115, which is bigger than 97. So five 23s do not fit. Go to M3.2.5.
   - **If you chose C:** 3 × 23 = 69 fits, but 97 − 69 = 28 is still more than 23. One more 23 fits. Go to M3.2.5.
   - **If you chose D:** 92 is the product, 4 × 23. The question asks how many 23s fit, so the answer is 4. Go to M3.2.5.

3. **Bring down the 7 next to the 9 to make 97.**

   The leftover 9 and the next digit, 7, make 97. Then you ask how many 23s fit into 97.

   - **If you chose A:** The digits come down in order, from the left. After 78, the next digit is the 7. The 0 comes after it. Go to M3.2.5.
   - **If you chose B:** The 9 is a leftover, not an answer digit. The answer digit was the 3 you wrote above the 8. Go to M3.2.5.
   - **If you chose C:** There are still digits in 7 870 to bring down. You stop only when none are left. Go to M3.2.5.

4. **342 remainder 4**

   The answer above the bracket is 342. The last leftover is 4. So 7 870 ÷ 23 = 342 remainder 4.

   - **If you chose A:** The 46 is the last product, 2 × 23. The remainder is the leftover after taking it away, which is 4. Go to M3.2.5.
   - **If you chose B:** This leaves out the remainder. 23 × 342 = 7 866, and 7 870 is 4 more. Go to M3.2.5.
   - **If you chose D:** You may have left out the last answer digit. Each round writes one digit above the bracket. Check by multiplying back: 23 × 34 = 782, far from 7 870. Go to M3.2.5.

5. **17, 34, 51, 68, 85**

   17, then 17 + 17 = 34, then 34 + 17 = 51, then 51 + 17 = 68, then 68 + 17 = 85.

   - **If you chose B:** You added 10 each time. For multiples of 17, add 17 each time. Go to M3.2.5.
   - **If you chose C:** You doubled each time. That is 1, 2, 4, 8 and 16 times 17. Multiples go up one time at a time. Go to M3.2.5.
   - **If you chose D:** This starts at 2 × 17. The list starts with 1 × 17, which is 17 itself. Go to M3.2.5.

6. **403**

   Start with 48. 4 × 12 = 48, so write 4 and nothing is left. Bring down the 3. 12 goes into 3 no times, so write 0. Bring down the 6. 12 goes into 36 three times. The answer is 403.

7. **268 remainder 14**

   34 × 268 = 9 112, and 9 126 − 9 112 = 14.

8. **17, 34, 51, 68 and 85**

   Start at 17 and add 17 each time: 17, 34, 51, 68, 85.

9. **689 remainder 6**

   9 goes into 62 six times, leaving 8. 9 goes into 80 eight times, leaving 8. 9 goes into 87 nine times, leaving 6. So the answer is 689 remainder 6.

M3.2.6. Order of operations

1. **17**

   Multiply first. 3 × 4 = 12. Then add. 5 + 12 = 17.

   - **If you chose A:** You multiplied everything: 5 × 3 × 4. The first sign is +, so add 5 to the product 3 × 4. Go to M3.2.6.
   - **If you chose B:** You worked from left to right: 5 + 3 = 8, then 8 × 4 = 32. Multiply first: 3 × 4 = 12. Go to M3.2.6.
   - **If you chose C:** You added all the numbers: 5 + 3 + 4. The × sign means multiply, and that comes before adding. Go to M3.2.6.

2. **12**

   Divide and multiply share a step, so work from left to right. 18 ÷ 3 = 6. Then 6 × 2 = 12.

   - **If you chose B:** You multiplied first: 3 × 2 = 6, then 18 ÷ 6 = 3. Multiplying and dividing share a step, so work from left to right. Go to M3.2.6.
   - **If you chose C:** You multiplied 18 × 3 × 2. The ÷ sign means divide: 18 ÷ 3 = 6 first. Go to M3.2.6.

3. **40**

   Brackets first. 6 + 2 = 8. Then 8 × 5 = 40.

   - **If you chose A:** You added all the numbers. The sign outside the brackets is ×. Work out the brackets, 8, then multiply by 5. Go to M3.2.6.
   - **If you chose B:** You ignored the brackets and multiplied first: 2 × 5 = 10, then 6 + 10. Brackets come first: 6 + 2 = 8. Go to M3.2.6.

4. **(2 + 3) × 4 = 20**

   The adding must happen first, so the brackets go round 2 + 3. Then 5 × 4 = 20.

   - **If you chose A:** Brackets round 3 × 4 change nothing, because multiplying already comes first. The answer stays 14. Go to M3.2.6.
   - **If you chose C:** Brackets round the whole calculation change nothing. The order inside is the same, so the answer is 14. Go to M3.2.6.

5. **(3 + 4) × 2 = 14**

   Each bag holds 3 + 4 = 7 beans. Two bags hold 7 × 2 = 14 beans. The brackets show that 3 + 4 is done first.

   - **If you chose B:** That is 3 loose beans and 4 bags of 2. Here every bag holds both white and black beans. Go to M3.2.6.
   - **If you chose C:** Only the white beans were doubled. The 4 black beans are in both bags too, so add 3 and 4 first. Go to M3.2.6.

6. **26**

   Multiply first. 6 × 3 = 18. Then 8 + 18 = 26.

7. **48**

   Brackets first. 7 + 5 = 12. Then 12 × 4 = 48.

8. **30**

   Divide first. 12 ÷ 3 = 4. Then work from left to right. 30 − 4 = 26. Then 26 + 4 = 30.

9. **247**

   Start with 59. 2 × 24 = 48 and 59 − 48 = 11. Bring down the 2 to make 112. 4 × 24 = 96 and 112 − 96 = 16. Bring down the 8 to make 168. 7 × 24 = 168. The answer is 247, with 0 left over.

M3.2.7. Estimate, then check

1. **50**

   52 is between 50 and 60. It is before the half way mark 55, so it rounds down to 50.

   - **If you chose A:** That is the number itself. Rounding changes it to a nearby number with zeros. Go to M3.2.7.
   - **If you chose B:** 52 is only 2 above 50 but 8 below 60. It is nearer to 50, so it rounds down. Go to M3.2.7.
   - **If you chose C:** 55 is the half way mark, not a rounded number. Rounding to one digit gives 50 or 60. Go to M3.2.7.

2. **400 × 50 = 20 000**

   389 rounds to 400 and 52 rounds to 50. 4 × 5 = 20, and three zeros go back. 400 × 50 = 20 000.

   - **If you chose A:** 389 is nearer to 400 than to 300, because it is past the half way mark 350. It rounds up. Go to M3.2.7.
   - **If you chose C:** 52 is about 50, not 5. You lost a zero, so the estimate is ten times too small. Go to M3.2.7.
   - **If you chose D:** The calculation is a multiplication. Round the numbers, but keep the × sign. Go to M3.2.7.

3. **Multiply 142 by 7, then add 6. The result should be 1 000.**

   142 × 7 = 994, and 994 + 6 = 1 000. That is the number she started with, so her division is right.

   - **If you chose A:** 142 × 7 = 994, not 1 000. The remainder must be added back to reach 1 000. Go to M3.2.7.
   - **If you chose B:** Dividing again does not undo the first calculation. The opposite of dividing is multiplying. Go to M3.2.7.
   - **If you chose C:** Adding 142 and 7 does not undo the division. Multiply 142 by 7, then add the 6. Go to M3.2.7.

4. **Yes, because 5 215 + 2 = 5 217.**

   745 × 7 = 5 215. Adding the remainder 2 gives 5 217, the number he started with.

   - **If you chose B:** The remainder 2 is the gap. 5 215 and the 2 left over make 5 217, so the numbers do match. Go to M3.2.7.
   - **If you chose C:** A remainder does not have to be 0. Many divisions leave something over. Go to M3.2.7.
   - **If you chose D:** Close is not enough for a check. The check must give the number exactly, and here it does after adding the 2. Go to M3.2.7.

5. **8 000 ÷ 20 = 400**

   7 870 rounds to 8 000 and 23 rounds to 20. 8 000 ÷ 20 is 800 ÷ 2, which is 400.

   - **If you chose A:** 23 is about 20, not 2. Using 2 makes the estimate ten times too big. Go to M3.2.7.
   - **If you chose B:** 7 870 is about 8 000, not 800. You lost a zero, so the estimate is ten times too small. Go to M3.2.7.

6. **24 000**

   612 rounds to 600 and 38 rounds to 40. 6 × 4 = 24, and three zeros go back. The estimate is 24 000.

7. **5 701**

   814 × 7 = 5 698. Adding the remainder, 5 698 + 3 = 5 701. This is the number that was divided.

8. **33**

   Multiply first. 4 × 6 = 24. Then 9 + 24 = 33.

M3.2.8. Choosing what to do

1. **144 ÷ 6 = 24**

   The total, 144, is known, and the bar is cut into 6 equal parts. From the total to one equal part, divide. 144 ÷ 6 = 24.

   - **If you chose A:** Adding makes the bar longer. We know the whole bar and want one part, so divide. Go to M3.2.8.
   - **If you chose B:** Multiplying makes a number bigger than 144, but one part cannot be bigger than the whole bar. Go to M3.2.8.
   - **If you chose C:** Taking away 6 does not cut the bar into 6 equal parts. Equal parts mean divide. Go to M3.2.8.

2. **6 × 24 = 144**

   The 6 equal parts are known and the total is wanted. Parts to total means multiply. 6 × 24 = 144.

   - **If you chose A:** That adds the number of parts to the size of one part. The whole bar has 6 parts of 24, so multiply. Go to M3.2.8.
   - **If you chose B:** That cuts one part of 24 into 6 smaller pieces. The question asks for the whole bar. Go to M3.2.8.

3. **15**

   First find all the beads. 3 × 24 = 72, and 72 + 18 = 90. Then share them. 90 ÷ 6 = 15.

   - **If you chose B:** You left out the 18 loose beads. All the beads are shared, so add them to the 72 first. Go to M3.2.8.
   - **If you chose C:** You divided only the 18 loose beads by 6. All the beads must be shared, so add first, then divide. Go to M3.2.8.
   - **If you chose D:** You multiplied the total by 6. Sharing between 6 friends means dividing the total by 6. Go to M3.2.8.

4. **13**

   The total length, 156 m, is known, and each part is 12 m. The number of parts is wanted, so divide. 156 ÷ 12 = 13.

   - **If you chose A:** Multiplying gives a number far bigger than the rope. Each piece is 12 m long, so there are fewer pieces than metres. Go to M3.2.8.
   - **If you chose C:** Taking 12 m off the rope gives the rope that is left after one piece. It does not count the pieces. Go to M3.2.8.
   - **If you chose D:** Adding makes the rope longer. Cutting it into equal pieces means dividing. Go to M3.2.8.

5. **45**

   Ana has 3 equal parts of 15. Parts to total means multiply. 3 × 15 = 45.

   - **If you chose A:** Taking away makes the number smaller. Ana has more stickers than Tariq, so the answer is more than 15. Go to M3.2.8.
   - **If you chose B:** Dividing finds a third of 15. "3 times as many" means Ana has 3 parts of 15, so multiply. Go to M3.2.8.
   - **If you chose C:** Adding 3 would give her only 3 more. "3 times as many" means 3 equal parts of 15. Go to M3.2.8.

6. **17**

   First find all the beads. 4 × 25 = 100, and 100 + 36 = 136. Then share them. 136 ÷ 8 = 17.

7. **26**

   Given away: 3 × 40 = 120. Left: 250 − 120 = 130. Each cousin: 130 ÷ 5 = 26.

8. **15 000**

   487 rounds to 500 and 31 rounds to 30. 5 × 3 = 15, and three zeros go back. The estimate is 15 000.

About this book

For learners who can work out 23 × 4 and 84 ÷ 4 on paper and know the times tables. In many schools this fits children of about 11 to 13, and older learners who never learned the written methods can start here too. Each chapter takes one or two short sessions. A learner can work alone: every exercise has its answer and the reason for it. Each chapter also has a page for the adult, with three questions to ask.

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