●●●○○ Level 3 · Mathematics M3.7

Statistics and chance

Collect and show data, find averages, and measure how likely things are.

Write a clear survey question, group the answers and draw bar charts and pie charts. Then find the mean, median, mode and range, compare two groups, and spot a chart that misleads. The last three chapters measure chance on a scale from 0 to 1, work out chances, and test them with coins and dice.

YOU ARE READY FOR THIS BOOK IF

You can draw a bar chart from a frequency table.

One step up	M4.6 Probability
This book	**M3.7 Statistics and chance**
One step down	M2.7 Tables and charts

Contents

M3.7.1 Collecting data

M3.7.2 Bar charts and pie charts

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M3.7.6 The language of chance

M3.7.7 Working out chances

M3.7.8 Experiments and fair games

Key words, A to Z

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M3.7.1 CHAPTER 1

Collecting data

> **Ten friends gave me ten different answers to my survey. What went wrong?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** I asked too few friends.
- ☐ **B** My question was not clear, so each friend answered a different question.
- ☐ **C** My friends did not tell the truth.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Tables and charts, M2.7.1. A tally and a frequency table are how the answers of a survey are counted.

BY THE END YOU CAN

- Write a clear question for a survey
- Group data into classes like 0 to 9 and 10 to 19
- Tell counted data from measured data

A **survey** asks many people the same question. The answers you collect are called **data**. Good data comes from a good question.

A good survey question asks about one thing. It means the same to everyone. Its answers can be counted. Give a choice for every answer, such as "other".

Question	Good?
How do you go?	No
Walk, bus or other?	Yes
Do you like it?	No

"How do you go?" is too open. "Do you like it?" does not say what "it" is. "Walk, bus or other?" gives a choice for every answer, and the answers can be counted.

Ana asks 20 children how many books they read last month. The answers run from 0 to 29. A list of 20 numbers is hard to read. So we sort the numbers into groups called **classes**.

DRAW IT FIRST

Sort 20 answers into classes

3 12 7 0 15 9 24 18 2 29
11 6 21 14 8 5 19 27 10 4

1 Write down all 20 answers. Each one is a number of books.

3 12 7 0 15 9 24 18 2 29
11 6 21 14 8 5 19 27 10 4

0 to 9	10 to 19	20 to 29
3 7	12 15	24 29
0 9	18 11	21 27
2 6	14 19	
8 5	10	
4		

2 Make a box for each class. Put each answer in the box where it fits.

3 12 7 0 15 9 24 18 2 29
11 6 21 14 8 5 19 27 10 4

0 to 9	10 to 19	20 to 29
3 7	12 15	24 29
0 9	18 11	21 27
2 6	14 19	
8 5	10	
4		
9	**7**	**4**

3 Count the answers in each box. These counts are the frequencies.

Books read	Children
0 to 9	9
10 to 19	7
20 to 29	4

9 + 7 + 4 = 20, so every answer is in exactly one class.

Good classes are usually the same width. The classes 0 to 9 and 10 to 19 both hold 10 whole numbers. There is no gap between classes, and they never overlap. So every answer fits one class only. The last class may be open, such as "10 or more".

WORKED EXAMPLE

Sipho asked 10 children how many books they read. The answers are 4, 13, 8, 21, 17, 2, 10, 25, 6, 19. Sort the answers into the classes 0 to 9, 10 to 19 and 20 to 29. How many are in each class?

1. Class 0 to 9: the answers 4, 8, 2 and 6 fit. That is 4.
2. Class 10 to 19: the answers 13, 17, 10 and 19 fit. That is 4.
3. Class 20 to 29: the answers 21 and 25 fit. That is 2.
4. Check: 4 + 4 + 2 = 10, the number of children asked.

Answer: The counts are 4, 4 and 2.

0 to 9	10 to 19	20 to 29
4 8 2 6	13 17 10 19	21 25
4	4	2

Sipho's 10 answers, sorted into their classes. The number under a box is how many answers it holds.

Data comes in two kinds. **Counted data** is found by counting, so it is a whole number: 3 books, 0 pets. **Measured data** is found with a ruler, a scale or a clock. It can be 142.5 cm or 7.25 s.

Counted	Measured
pets owned	height
goals scored	time to run
books read	mass of a bag

Ask: did I count it, or measure it? Counted data skips between whole numbers. Measured data can be any size in between.

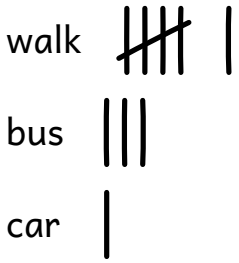
Classes like 0 to 9 and 10 to 19 work for counted data. A measured 9.5 cm would fit neither of them. So measured classes are written like 10 cm up to 20 cm. The 20 cm goes in the next class.

SEE IT THREE WAYS

With things

Ana asks 10 children how they get to school. She makes one mark for each answer. Every fifth mark crosses the four before it.

As a picture



In symbols

walk 6, bus 3, car 1. 6 + 3 + 1 = 10 children.

TRY IT

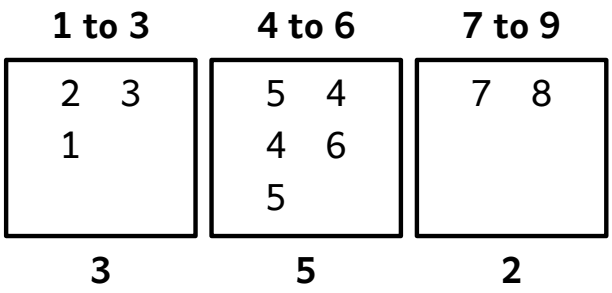
A survey at home

You need: paper and a pencil, ten people to ask, or ten answers you can find.

1. Write the question: "How many people live in your home?"
2. Ask ten people and write each answer down.
3. Make three classes: 1 to 3, 4 to 6 and 7 to 9. If a home has 10 people or more, add a class "10 or more".
4. Put each answer in its class and count.

What you will see: Every answer fits one class, and the counts add up to 10. If they do not, an answer fits no class, or a class has a gap or an overlap.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~Classes can overlap, like 0 to 10, 10 to 20 and 20 to 30.~~

Why it feels right: The numbers 10 and 20 look like good places to stop and start.

The test that breaks it: Put the answer 10 in the classes 0 to 10 and 10 to 20. It fits both, so you do not know which count to add it to.

What is true: Every answer must fit one class only. Use 0 to 9, then 10 to 19, then 20 to 29.

KEY WORDS

survey

asking many people the same question.

data
the facts and numbers you collect.

class
one group of values, such as 10 to 19.

counted data
data found by counting, always a whole number.

measured data
data found with a ruler, scale or clock.

BACK TO YOUR GUESS

The answer is B: My question was not clear, so each friend answered a different question.

A survey question must mean the same thing to everyone. "How do you travel?" gets answers like "fast" and "in the morning". Give choices: walk, bus, car or another way. Then the answers can be counted.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Tariq wants to know how children travel to school. Which is the clearest survey question?

   A How do you usually get to school: walk, bicycle, bus, car or other?

   B Do you like going to school?

   C How do you travel?

   Answer at the back of the book.

2. The answers in a survey run from 0 to 29. Which classes are set up correctly?

   A 0 to 10, 10 to 20, 20 to 29

   B 0 to 9, 11 to 19, 20 to 29

   C 0 to 9, 10 to 19, 20 to 29

   Answer at the back of the book.

3. Which of these is measured data?

   A The number of pets in a home

   B The mass of a bag of rice

   C The number of players on a team

   Answer at the back of the book.

4. The classes are 0 to 9, 10 to 19 and 20 to 29. In which class does the answer 19 go?

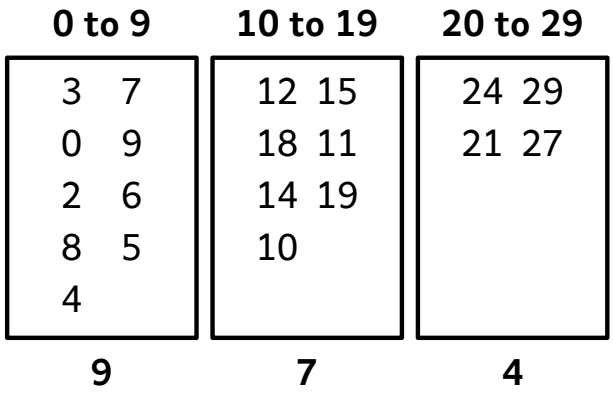
   A 10 to 19

   B 0 to 9

   C 20 to 29

   Answer at the back of the book.

5. The boxes show the books read by 20 children. How many children read 10 to 19 books?



A 19 children

B 9 children

C 7 children

Answer at the back of the book.

6. Hana asked 8 children how many books they read: 5, 12, 27, 9, 14, 3, 21, 17. How many of them read 10 to 19 books?

Answer at the back of the book.

7. A frequency table shows 6, 11 and 8 children in three classes. How many children were asked?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A clear survey question asks about one thing and has answers you can count. Classes have the same width, no gap and no overlap. Counted data is whole numbers. Measured data can be any size.

Draw: Draw three boxes named 0 to 9, 10 to 19 and 20 to 29. Write six answers in the right boxes.

Bar charts and pie charts

> **Twelve of 36 children walk to school. How big is the walk sector of a pie chart?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 12°
- ☐ **B** 120°
- ☐ **C** 36°

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Tables and charts, M2.7.3. A bar chart has a scale in equal steps. Each bar stands on zero.
- Level 2, Shapes and angles, M2.5.4. A pie chart uses degrees: a whole turn is 360° and a right angle is 90°.

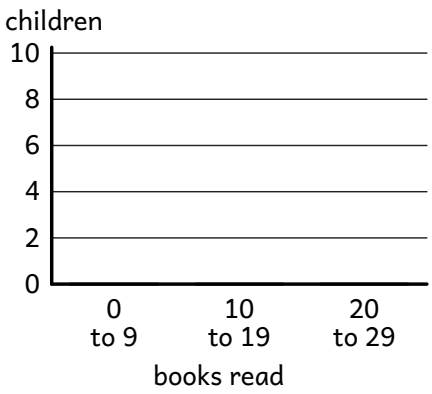
BY THE END YOU CAN

- Draw a bar chart for grouped data
- Draw a pie chart by working out the angles
- Read a pie chart to find fractions of the whole

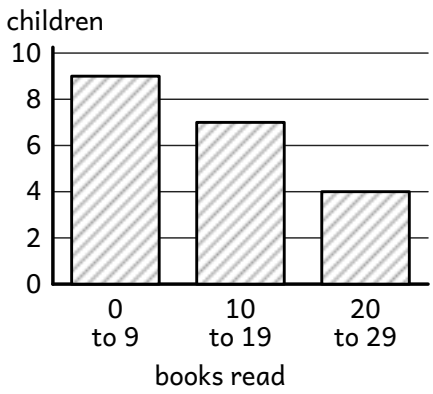
Last chapter you sorted answers into classes and counted them. Now we draw the counts. A **bar chart** shows how many are in each class. A **pie chart** shows how a whole is shared out.

DRAW IT FIRST

Draw a bar chart for the classes

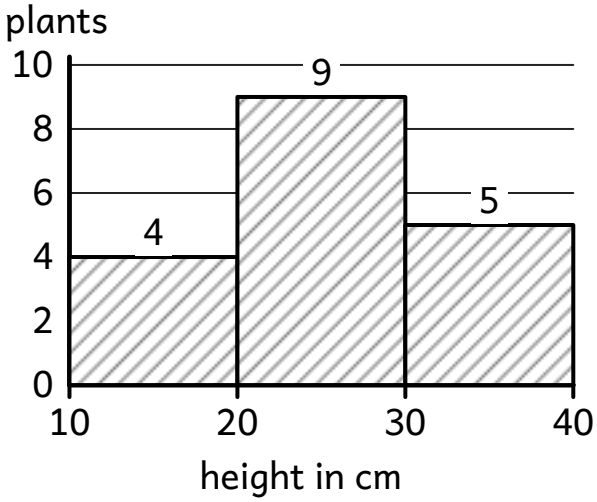


1 Draw the axes. Put the scale on the side, going up in equal steps from 0. Write each class under its place.



2 Draw a bar for each class, as high as its count. Leave a gap between the bars.

Why the gap? Counted data jumps from 9 to 10 with nothing between. The gap shows that. Bars for measured classes touch, because measured data has no jump. Always start the scale at 0, and make every step the same size.



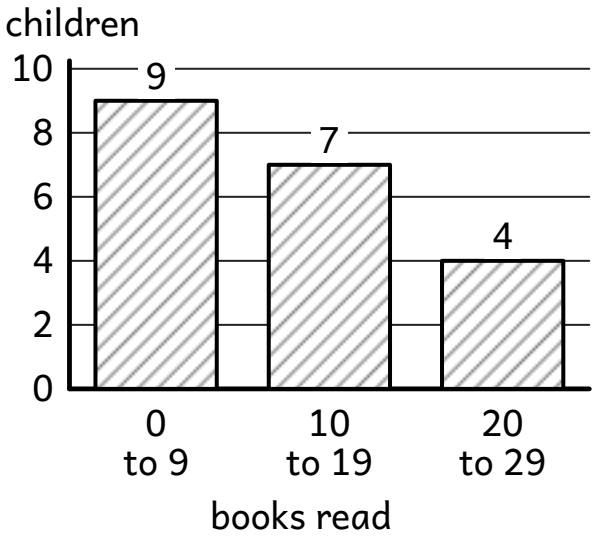
Heights of 18 plants. A plant of 20 cm goes in the second bar. The bars touch, so no height falls in a gap.

WORKED EXAMPLE

A frequency table shows 9, 7 and 4 children in three classes. The classes are 0 to 9, 10 to 19 and 20 to 29. Choose a scale for the bar chart. How high is each bar?

1. The biggest count is 9, so a scale up to 10 is enough.
2. Go up in steps of 2. From 0 to 10 that is 10 ÷ 2 = 5 steps, so the chart is not crowded.
3. The bars reach 9, 7 and 4. The bar for 9 stops halfway between the lines for 8 and 10.

Answer: The bars are 9, 7 and 4 high.



The same bar chart, with each count written on top of its bar.

A bar chart is good for comparing classes. A pie chart is good when the parts make one whole, such as how 36 children travel. A whole turn is 360°. Each child gets the same share of that turn.

DRAW IT FIRST

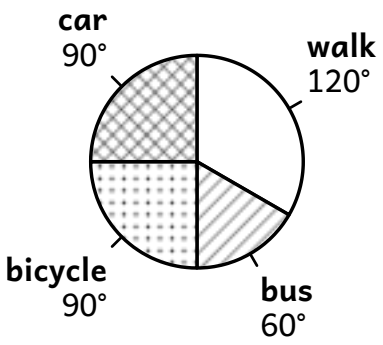
Draw a pie chart for 36 children

Way	Children
walk	12
bus	6
bicycle	9
car	9

1 Count each group and add them: 12 + 6 + 9 + 9 = 36 children.

Way	Children	Angle
walk	12	120°
bus	6	60°
bicycle	9	90°
car	9	90°

2 One child is 360 ÷ 36 = 10°. Multiply each count by 10 to get its angle.



3 Draw a circle. Draw each sector with its angle. Name every sector.

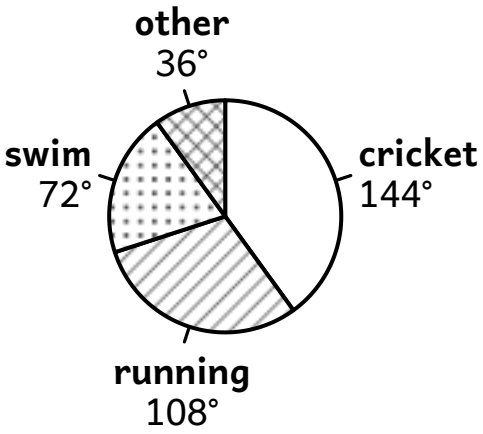
The rule is: angle = count ÷ total × 360°. A **sector** is one slice of the circle. Each sector has its own pattern and its own name. So the chart works on a black and white copy.

WORKED EXAMPLE

Kofi asked 20 children for their favourite sport. 8 chose cricket, 6 running, 4 swim and 2 other. Work out the angle of each sector.

1. There are 8 + 6 + 4 + 2 = 20 children in all.
2. One child is 360 ÷ 20 = 18°.
3. Cricket is 8 × 18 = 144°. Running is 6 × 18 = 108°.
4. Swim is 4 × 18 = 72°. Other is 2 × 18 = 36°.
5. Check: 144 + 108 + 72 + 36 = 360.

Answer: 144°, 108°, 72° and 36°



Kofi's pie chart. The four angles add up to 360°, one whole turn.

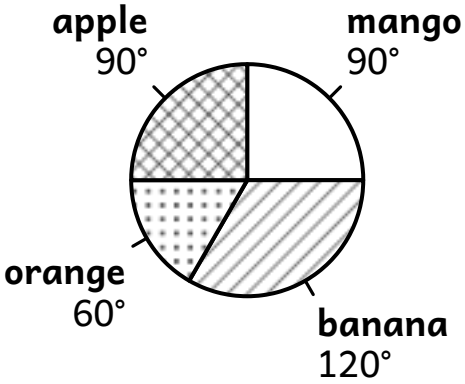
You can also read a pie chart. A sector's angle out of 360° is its fraction of the whole. A right angle of 90° is $^1/_4$ of the circle. A straight angle of 180° is $^1/_2$.

WORKED EXAMPLE

The pie chart shows the favourite fruit of 24 children. The mango sector is 90°. What fraction of the children chose mango, and how many children is that?

1. One child is 360 ÷ 24 = 15°.
2. The mango sector is 90°, so 90 ÷ 15 = 6 children.
3. As a fraction: 90 out of 360 is $^1/_4$.
4. Check: $^1/_4$ of 24 children is 24 ÷ 4 = 6.

Answer: $^1/_4$, which is 6 children



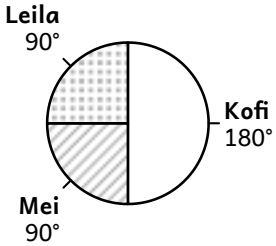
Favourite fruit of 24 children. The mango sector is a quarter of the circle.

SEE IT THREE WAYS

With things

Ana shares a round bread. Kofi gets half. Mei and Leila get a quarter each. The pieces meet at the middle.

As a picture



In symbols

Half of 360° is 180°. A quarter of 360° is 90°. 180° + 90° + 90° = 360°.

TRY IT

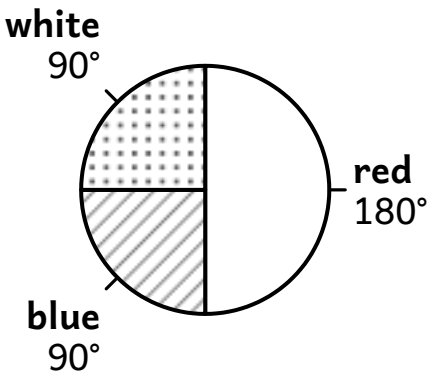
Fold a pie chart

You need: a round piece of paper, a pencil.

1. Fold the paper circle in half. Fold it in half again. Fold it once more.
2. Open it. The folds make 8 equal sectors.
3. Each sector is 360 ÷ 8 = 45°. Write red in 4 sectors, blue in 2 and white in 2.
4. Check that the angles add up to 360°.

What you will see: Red takes half the circle, 180°. Blue and white take 90° each. Each fold made a sector the same size.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The angle of a sector is the same as its count. 12 children make a sector of 12°.~~

Why it feels right: The count is the number you can see, so it feels natural to use it.

The test that breaks it: Add up the angles that way: 12 + 6 + 9 + 9 = 36. A pie chart must add up to 360°, so something is wrong.

What is true: The angle is count ÷ total × 360. Here that is 12 ÷ 36 × 360 = 120°.

KEY WORDS

bar chart
a chart with one bar for each class, as high as its count.

pie chart
a circle cut into sectors, one for each part of a whole.

sector
one slice of a circle, drawn with an angle.

BACK TO YOUR GUESS

The answer is B: 120°

The whole circle is 360° and there are 36 children in all. One child is 360 ÷ 36 = 10°. So 12 children is 12 × 10 = 120°.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Mei asked 24 children for their favourite food. 6 chose rice. What is the angle of the rice sector?

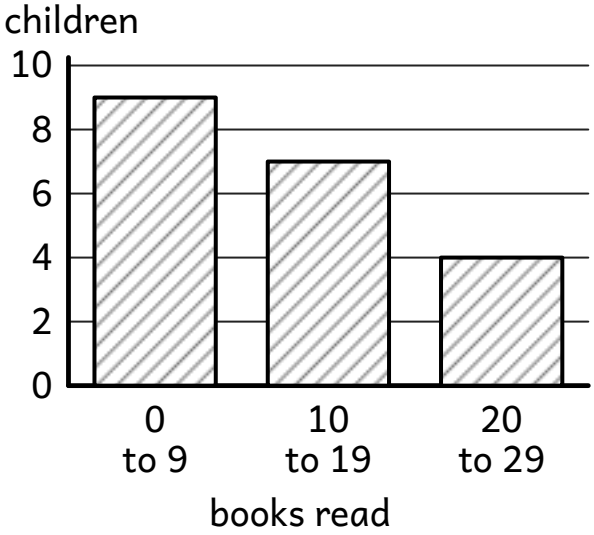
   A 90°

   B 15°

   C 6°

   Answer at the back of the book.

2. Which class has the most children in this bar chart?



A 20 to 29

B 9

C 0 to 9

Answer at the back of the book.

3. Which bar chart should have *a gap* between its bars?

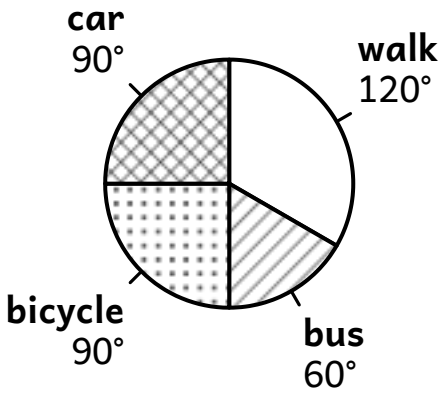
A A chart of the heights of plants in cm, grouped into classes

B A chart of the number of books each child read, grouped into classes

C Neither chart

Answer at the back of the book.

4. In the travel pie chart the walk sector is 120°. What fraction of the children walk?



A $^1/_3$

B $^1/_2$

C $^1/_{12}$

Answer at the back of the book.

5. Why must the angles of a pie chart always add up to 360°?

A Because every sector is a right angle.

B Because there are always 360 children.

C Together the sectors make one whole turn of the circle.

Answer at the back of the book.

6. In the travel pie chart, the bus sector is 60°. How many of the 36 children take the bus?

A 60 children

B 6 children

C 10 children

Answer at the back of the book.

7. Priya asked 30 children which game they like best. 5 chose football. What is the angle of the football sector?

Answer at the back of the book.

8. A pie chart has a sector of 45°. What fraction of the circle is it?

   Answer at the back of the book.

9. **From an earlier chapter**

   Review: classes are 0 to 9, 10 to 19 and 20 to 29. In which class does the answer 14 go?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A bar chart starts at 0, has equal steps and a gap between bars. A pie chart sector is count ÷ total × 360°. Sector angles add up to 360°. A sector of 90° is a quarter.

Draw: Draw a circle cut into sectors of 180°, 90° and 90°. Name each sector and write its angle.

Mean, median and mode

> **Five friends have 2, 3, 3, 4 and 8 pencils. Shared out equally, how many pencils does each friend get?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** 4 pencils
- ☐ **B** 3 pencils
- ☐ **C** 8 pencils

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Times tables and sharing, M2.3.3. The mean is a fair share: the total shared equally.

BY THE END YOU CAN

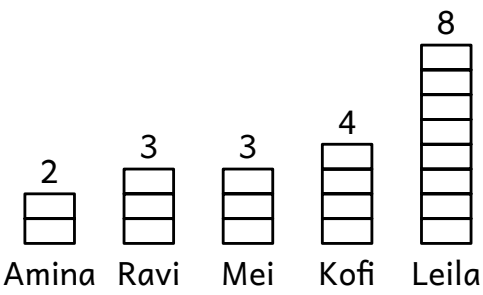
- Find the mean, median and mode of a set of numbers
- Find the range to see how spread out the data is

A list of numbers can be long. An ***average*** is one number that stands for the whole list. There are three common averages: the mean, the median and the mode.

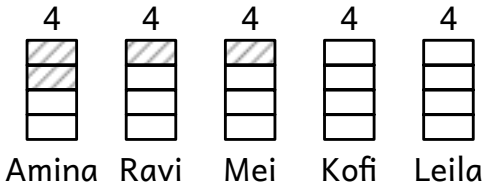
Five friends have 2, 3, 3, 4 and 8 pencils. Show each friend as a tower of cubes, one cube for each pencil. Share the cubes out until all towers are level. That height is the **mean**.

DRAW IT FIRST

Level the towers to find the mean



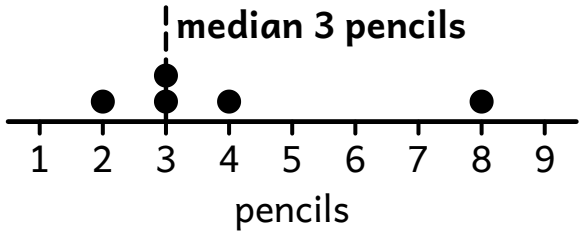
1 Build a tower for each friend. The tallest tower has 8 cubes and the shortest has 2.



2 Move cubes from the tall towers to the short ones until all five towers are level. Each tower has 4 cubes.

To find the mean without cubes, add all the numbers. Then divide by how many there are. Here 20 ÷ 5 = 4. The mean is the total shared equally.

The **median** is the middle number when the list is in order. For 2, 3, 3, 4, 8 the middle one is 3. The **mode** is the number that appears most often. Here the mode is also 3. If no number appears more than once, there is no mode. If two numbers tie, both are modes: 1, 1, 2, 2 has the modes 1 and 2.



The dots stand in order along the line. Two values come before the middle one and two come after it. The middle value is the second dot at 3.

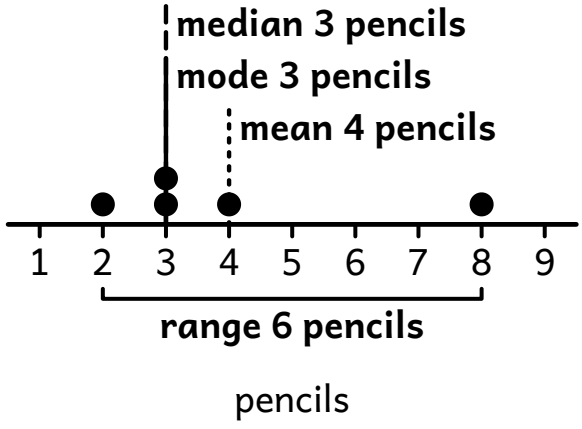
The **range** is the largest number take away the smallest. It is not an average. It tells you how spread out the numbers are.

WORKED EXAMPLE

Find the mean, median, mode and range of the pencils 2, 3, 3, 4 and 8.

1. Add the five numbers: 2 + 3 + 3 + 4 + 8 = 20.
2. Mean: share the total between 5 friends. 20 ÷ 5 = 4.
3. Median: the numbers are already in order. The middle one is 3.
4. Mode: 3 appears twice, more than any other number. The mode is 3.
5. Range: the largest is 8 and the smallest is 2. 8 − 2 = 6.

Answer: Mean 4, median 3, mode 3 and range 6.



The mean, median and mode are not always the same. Here the mean is 4 and the median and mode are 3.

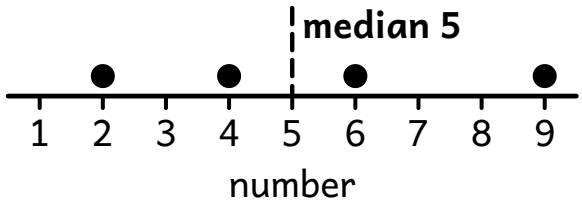
What if the list has an even number of values? Then there are two middle numbers. The median is halfway between them.

WORKED EXAMPLE

Find the median of 6, 2, 9 and 4.

1. Put them in order: 2, 4, 6, 9.
2. Four numbers have two middle ones, 4 and 6. Add them: 4 + 6 = 10.
3. Halfway between them is half of the sum: 10 ÷ 2 = 5.

Answer: 5



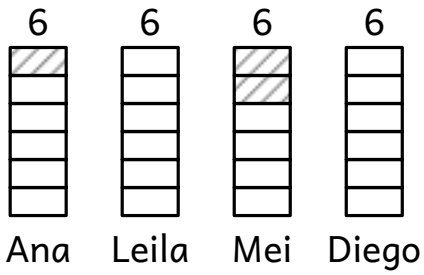
The median 5 is between the two middle dots. No dot sits on it.

WORKED EXAMPLE

The mean of 4 numbers is 6. Three of them are 5, 7 and 4. Find the fourth number.

1. The total of 4 numbers with mean 6 is 6 × 4 = 24.
2. The three known numbers add up to 5 + 7 + 4 = 16.
3. The fourth number is 24 − 16 = 8.
4. Check: the mean of 5, 7, 4 and 8 is 24 ÷ 4 = 6.

Answer: 8



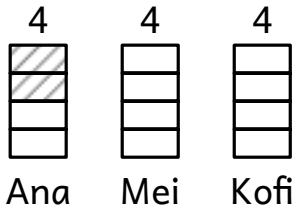
Towers of 5, 7, 4 and 8 cubes, shared out. Each ends with 6 cubes, which is the mean.

SEE IT THREE WAYS

With things

Ana, Mei and Kofi have 2, 4 and 6 beans. Put all the beans in one pile. Deal them out one at a time until the pile is gone.

As a picture



In symbols

2 + 4 + 6 = 12. 12 ÷ 3 = 4. The mean is 4.

TRY IT

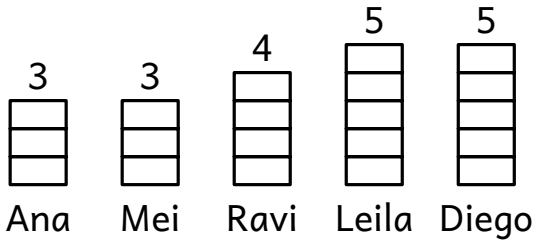
The mean length of names

You need: paper and a pencil, five names, such as your family or friends.

1. Write five names. Count the letters in each name.
2. Draw a tower of squares for each name, one square for each letter.
3. Add up all the squares. Divide by 5.
4. Say what number of letters a name has on average.

What you will see: The names Ana, Mei, Ravi, Leila and Diego have 3, 3, 4, 5 and 5 letters. That is 20 letters in all. The mean is 20 ÷ 5 = 4 letters.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~The median is the middle number of the list as it is written.~~

Why it feels right: The word "middle" makes you look at the middle of the page.

The test that breaks it: Take 8, 2, 9, 4, 6. The middle one as written is 9. Now put them in order: 2, 4, 6, 8, 9. The middle one is 6.

What is true: Put the numbers in order first. The median is the middle of the ordered list.

KEY WORDS

average
one number that stands for a whole list.

mean
the total shared equally: add them up, then divide by how many.

median
the middle number when the list is in order.

mode
the number that appears most often. A list can have no mode, or two.

range
the largest number take away the smallest.

BACK TO YOUR GUESS

The answer is A: 4 pencils

The five friends have 2 + 3 + 3 + 4 + 8 = 20 pencils. Shared equally between 5 friends, each gets 20 ÷ 5 = 4 pencils. That number is called the mean.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Find the mean of 2, 3 and 10.

   A 3

   B 15

   C 5

   Answer at the back of the book.

2. Find the median of 8, 2, 9, 4 and 6.

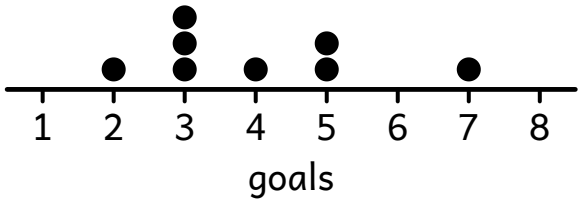
   A 9

   B 6

   C 5.8

   Answer at the back of the book.

3. The dot plot shows the goals scored by a team in 8 matches. What is the mode?



   A 3 goals

   B 7 goals

   C 5 goals

   Answer at the back of the book.

4. Find the range of 12, 5, 9, 20 and 7.

   A 20

   B 25

   C 15

   Answer at the back of the book.

5. The mean of 3 numbers is 6. Two of the numbers are 4 and 5. What is the third number?

   A 6

   B 9

   C 15

   Answer at the back of the book.

6. Find the mean of 3, 7, 4 and 6.

   Answer at the back of the book.

7. Find the median of 9, 3, 7, 12 and 5.

   Answer at the back of the book.

8. **From an earlier chapter**

   Review: Hana asked 40 children which game they like best. 8 chose football. What is the angle of the football sector in a pie chart?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Mean: add up, then divide by how many. Median: the middle of the ordered list. Mode: the most common, if any. Range: largest take away smallest. The range shows the spread.

Draw: Draw towers of 2, 4 and 6 cubes. Next to them draw three towers of 4 cubes. Write mean = 4.

M3.7.4 CHAPTER 4

Comparing two groups

Two teams both score 4 goals a game on average. Are the teams the same?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** No. The means are equal, but one team may be much less steady than the other.
- ☐ **B** Yes. The same mean means the two teams are exactly the same.
- ☐ **C** No. The team that scored first is better.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Tables and charts, M2.7.3. Comparing groups starts with reading a chart that shows each group.

BY THE END YOU CAN

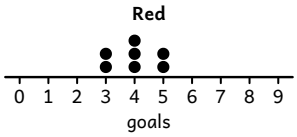
- Compare two sets of data using an average and the range
- Choose the best average for a set of data

To compare two groups, use two numbers. An average tells you what is typical. The range tells you how **spread** out the data is. A small range means the data is steady.

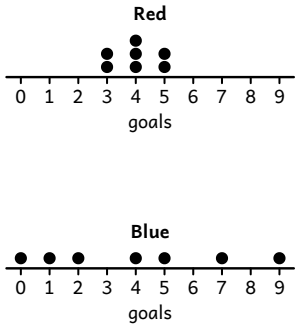
Teams Red and Blue each played 7 matches. We put their goals on the same scale, so the two pictures can be compared.

DRAW IT FIRST

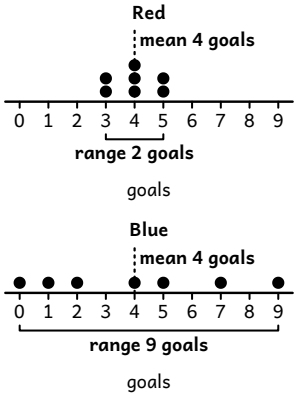
Compare team Red and team Blue



1 Draw a dot plot for team Red. Put one dot for each match.



2 Draw team Blue on the same scale from 0 to 9.



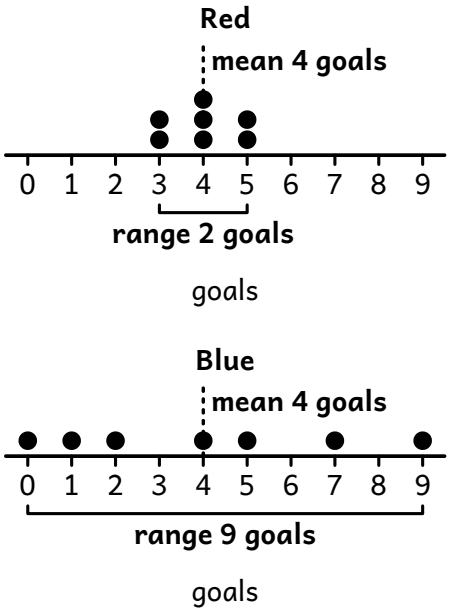
3 Mark the mean and the range on each plot.

WORKED EXAMPLE

Red scored 3, 3, 4, 4, 4, 5 and 5 goals. Blue scored 0, 1, 2, 4, 5, 7 and 9 goals. Compare the two teams.

1. Red: 3 + 3 + 4 + 4 + 4 + 5 + 5 = 28. The mean is 28 ÷ 7 = 4.
2. Blue: 0 + 1 + 2 + 4 + 5 + 7 + 9 = 28. The mean is 28 ÷ 7 = 4.
3. The range of Red is 5 − 3 = 2.
4. The range of Blue is 9 − 0 = 9.

Answer: The means are equal, but Red is steadier. The ranges are 2 and 9.



Same mean, different range. Red is steady. Blue is up and down.

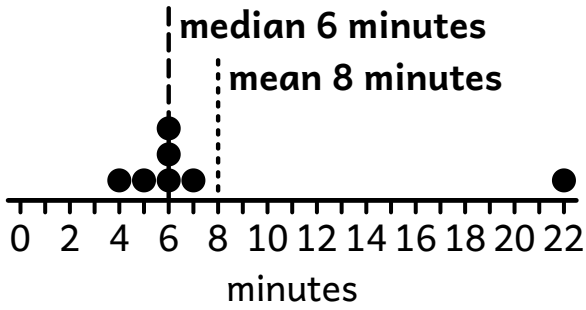
Which average should you use? The mean uses every value, so one very big or very small value can pull it. An odd value like that is called an **outlier**.

WORKED EXAMPLE

Seven children walk to school. Their times in minutes are 4, 5, 6, 6, 6, 7 and 22. Which average describes a typical walk best?

1. Add the times: 4 + 5 + 6 + 6 + 6 + 7 + 22 = 56.
2. Mean: 56 ÷ 7 = 8.
3. Median: in order, the middle (4th) time is 6.
4. The walk of 22 minutes is an outlier. It pulls the mean up. The median stays close to most children.

Answer: The median, 6 minutes, describes a typical walk better.



Six dots are close together. One dot, the outlier, is far away. The mean is dragged towards it.

Average	Best when
mean	no outliers
median	an outlier
mode	most common

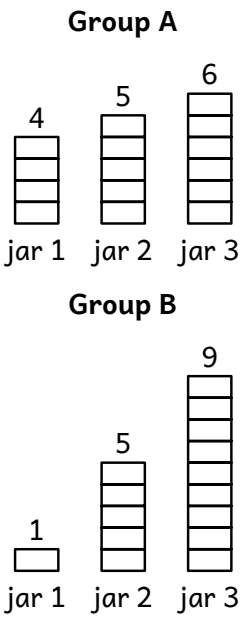
Mode is the only average that works for words, such as the colour chosen most often.

SEE IT THREE WAYS

With things

Two groups of three jars hold beans. Group A has 4, 5 and 6 beans. Group B has 1, 5 and 9 beans.

As a picture



In symbols

Both means are 15 ÷ 3 = 5. The ranges are 6 − 4 = 2 and 9 − 1 = 8.

TRY IT

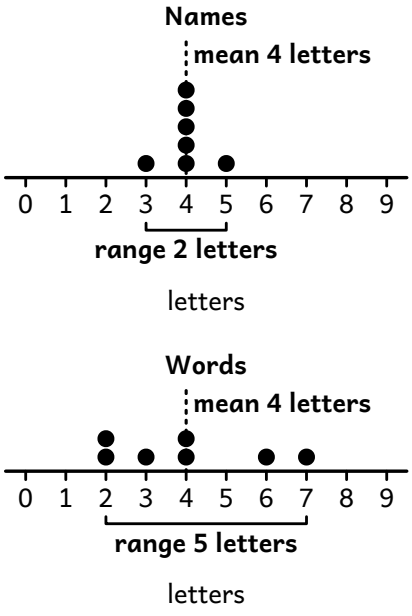
Names against words

You need: paper and a pencil, a page of writing.

1. Write the first names of 7 people. Count the letters in each name.
2. Pick 7 words from a page. Count the letters in each word.
3. Find the mean and the range of each group.
4. Say which group is more spread out.

What you will see: Take seven first names with 3, 4, 4, 4, 4, 4 and 5 letters. The mean is 4 and the range is 2. Seven words with 2, 2, 3, 4, 4, 6 and 7 letters also have a mean of 4. Their range is 5.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~If two groups have the same mean, the groups are the same.~~

Why it feels right: The mean is one number that feels like the whole story.

The test that breaks it: Red and Blue both have a mean of 4 goals. Red never scored more than 5. Blue once scored 0 and once scored 9.

What is true: The mean says what is typical. The range says how spread out the data is. Use both.

KEY WORDS

spread
how far apart the values are, shown by the range.

outlier
a value far from the rest.

BACK TO YOUR GUESS

The answer is A: No. The means are equal, but one team may be much less steady than the other.

The mean says what is typical. It does not say how spread out the scores are. Red scored 3 to 5 goals every match. Blue scored from 0 to 9. To compare groups, use an average and the range.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. Group A and group B both have a mean of 10. The range of A is 3. The range of B is 12. Which is true?

   A Group A is more spread out.

   B The two groups are exactly the same.

   C Group B is more spread out.

   Answer at the back of the book.

2. Six children read for 5, 6, 6, 6, 7 and 8 minutes. A seventh reads for 46 minutes. Which average describes a typical reading time best?

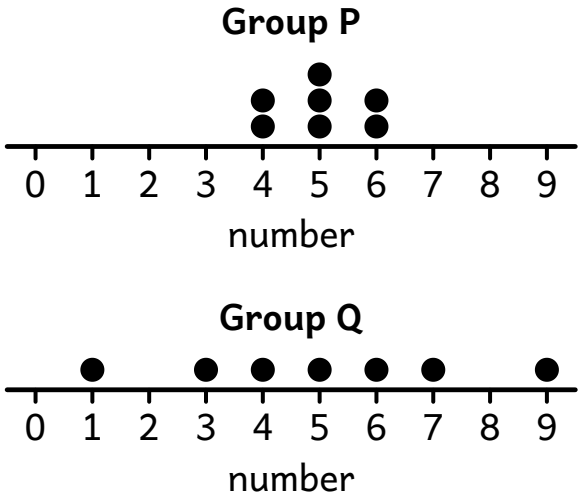
   A The mean, 12 minutes

   B The median, 6 minutes

   C The range, 41 minutes

   Answer at the back of the book.

3. Group P and group Q both have a mean of 5. Which group is more spread out, and what is its range?



   A Group Q, range 8

   B Group P, range 2

   C Group Q, range 9

   Answer at the back of the book.

4. A school wants to know which shoe size most children wear. Which average should it use?

   A The mean

   B The median

   C The mode

   Answer at the back of the book.

5. Team X has a mean of 6 goals a match and a range of 2. Team Y has a mean of 6 goals a match and a range of 7. Which team scores more steadily?

   A Team Y

   B Team X

   C Neither, because the means are the same

   Answer at the back of the book.

6. Both groups have a mean of 5. Group A is 4, 5 and 6. Group B is 1, 5 and 9. What is the range of the group that is more spread out?

   Answer at the back of the book.

7. Team A scores a mean of 7 goals a match. Team B scores a mean of 5. How many more goals a match does Team A score on average?

   Answer at the back of the book.

8. **From an earlier chapter**

   Review: find the median of 12, 4, 8 and 6.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Compare groups with an average and the range. The same mean can hide a different spread. An outlier pulls the mean, so use the median. Use the mode for the most popular.

Draw: Draw two dot plots on one scale. Give them the same mean but different ranges. Write both ranges.

M3.7.5 CHAPTER 5

Line graphs and honest charts

> **Two graphs show the same numbers. Why does one line climb so much more steeply?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** The steeper graph has bigger numbers.
- ☐ **B** One of the graphs has wrong numbers.
- ☐ **C** The steeper graph starts its scale above zero, so part of the graph is cut away.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Tables and charts, M2.7.6. A line graph shows change over time, with each point read from the scale.

BY THE END YOU CAN

- Draw a line graph for data that changes over time
- Spot a chart that misleads by not starting at zero

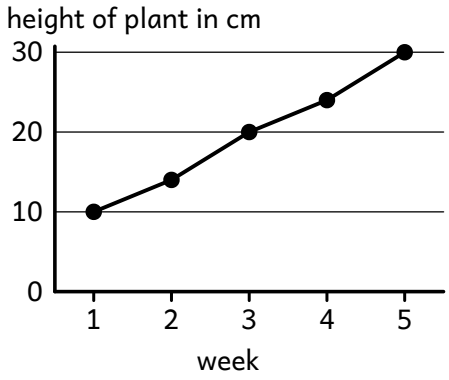
A **line graph** shows how something changes over time, such as the height of a plant each week. Each point is a measurement. The straight lines join the points so you can see the trend.

DRAW IT FIRST

Draw a line graph of a growing plant

Week	Height in cm
1	10
2	14
3	20
4	24
5	30

1 Write the measurements in a table: the week and the height.



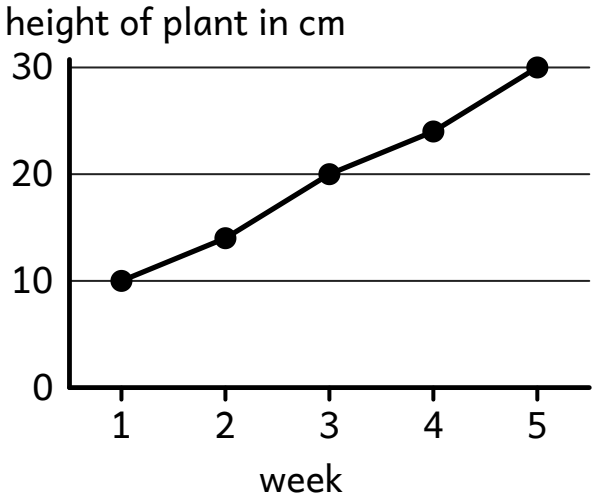
2 Mark a point for each week at its height. Join the points with straight lines, from left to right.

WORKED EXAMPLE

A plant is measured each week. The heights in cm are 10, 14, 20, 24 and 30. Draw a line graph. How much did the plant grow from week 1 to week 5?

1. The tallest height is 30 cm. A scale from 0 to 30 in steps of 10 is enough.
2. Put the weeks 1 to 5 along the bottom. Mark a point for each height. Week 3 is at 20.
3. Join the points with straight lines, from left to right.
4. Read the graph. From week 1 to week 5 the plant grew 30 − 10 = 20 cm.

Answer: 20 cm



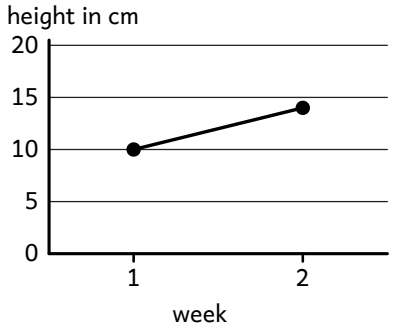
The line goes up every week, so the plant grows every week.

SEE IT THREE WAYS

With things

A bean plant is 10 cm tall in week 1 and 14 cm tall in week 2. Hold a ruler beside it each week and write down the height.

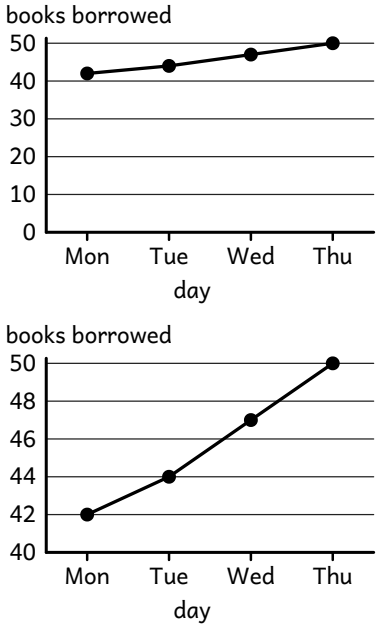
As a picture



In symbols

Week 1: 10 cm.
Week 2: 14 cm. It grew 14 − 10 = 4 cm.

Now look at a library. It lends 42, 44, 47 and 50 books on four days. Here are the same numbers on two graphs.



Same numbers. The first scale starts at 0 and the second at 40. The second line climbs five times as far.

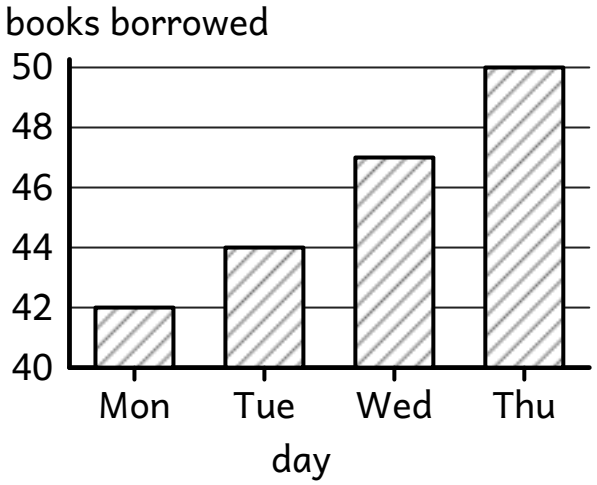
In the second graph the part of the scale below 40 is cut away. The line looks as if the library lends far more books each day. But it lends only 8 more on Thursday than on Monday. A bar chart must start its scale at 0, because the height of a bar is its count. A line graph that starts above 0 must say so with a zigzag.

WORKED EXAMPLE

A bar chart shows 42 books on Monday and 50 books on Thursday. Its scale starts at 40. How many more books were borrowed on Thursday? Is the chart fair?

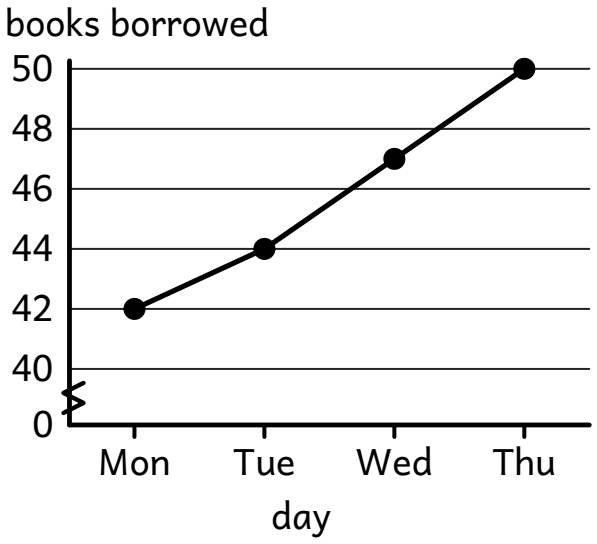
1. Read the bars: Monday is 42 and Thursday is 50.
2. Take away: 50 − 42 = 8 more books on Thursday.
3. The scale starts at 40, so every bar is cut. The Thursday bar looks 5 times as tall as the Monday bar. But 50 is not 5 times 42.
4. A fair chart starts at 0. Then the bars are 42 and 50 tall and look nearly the same.

Answer: 8 more books. The chart is not fair because the scale starts at 40.



The scale starts at 40. Thursday's bar looks 5 times as tall as Monday's, yet it holds only 8 more books.

Sometimes a graph must be cut, because all the numbers are large and close together. An honest graph then draws a zigzag on the axis. The zigzag says: part of the scale is missing.



The zigzag near the bottom of the side axis is a warning. This scale does not start at zero.

TRY IT

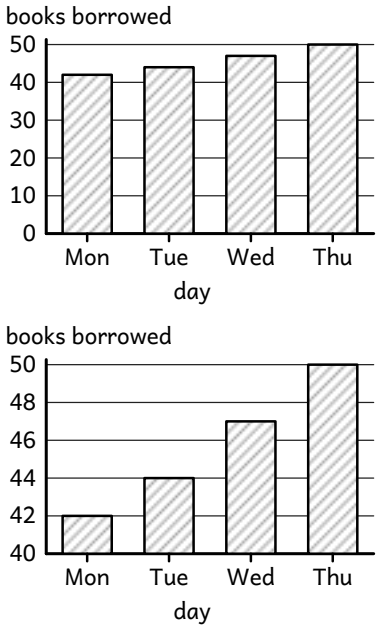
Draw it twice

You need: squared paper or plain paper, a pencil and a ruler.

1. Take the numbers 42, 44, 47 and 50.
2. Draw a bar chart with a scale from 0 to 50 in tens.
3. Draw a second bar chart of the same numbers, with the scale from 40 to 50 in twos.
4. Hold the two drawings side by side. Which one makes the rise look bigger?

What you will see: The bars on the first chart are nearly the same height. The bars on the second chart climb steeply. The numbers did not change. Only the scale did.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~A steeper line always means a bigger change.~~

Why it feels right: A steep line looks like a lot of change, and the picture is what we notice first.

The test that breaks it: Look at the scale of each graph. Both line graphs show the same 8 books of change. One starts at 0 and the other at 40.

What is true: Read the numbers on the axis. Where a graph starts and how big each step is decide how steep a line looks.

KEY WORDS

line graph
a graph with points joined by lines, for data that changes over time.

scale
the numbers along an axis, with equal steps.

misleading
giving a wrong picture, even if the numbers are right.

BACK TO YOUR GUESS

The answer is C: The steeper graph starts its scale above zero, so part of the graph is cut away.

Both graphs show the same numbers. The steep one starts its scale at 40 instead of 0, so small changes look big. Always read the numbers on the axis before you trust a picture.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A bar chart has a scale that starts at 40 and has no zigzag. Why can this be misleading?

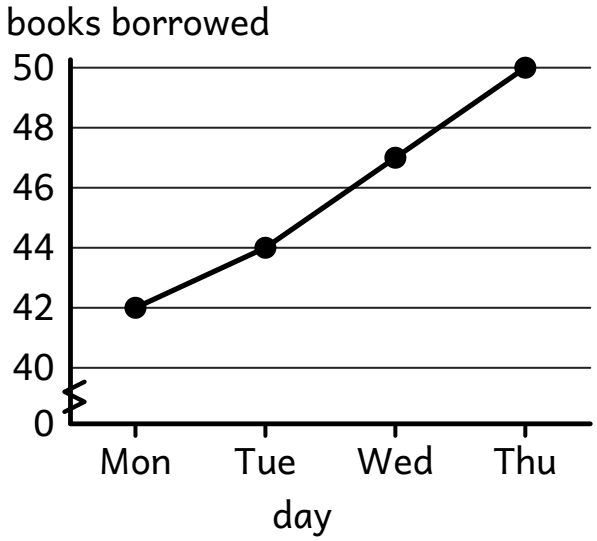
   A It makes small differences between the bars look big.

   B It adds 40 to every number.

   C It makes every bar the same height.

   Answer at the back of the book.

2. Read the line graph. The scale goes up in steps of 2. How many books were borrowed on Wednesday?



   A 46 books

   B 47 books

   C 48 books

   Answer at the back of the book.

3. Mei measures the temperature each hour through a day. Which graph is best?

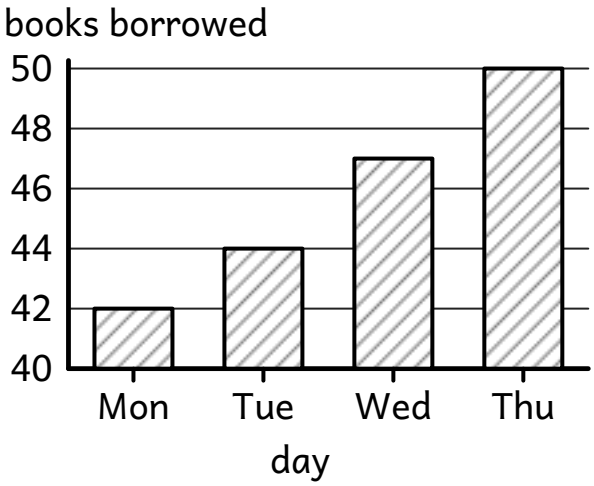
   A A bar chart of classes

   B A pie chart

   C A line graph

   Answer at the back of the book.

4. Thursday's bar looks 5 times as tall as Monday's. Is Thursday's book count 5 times Monday's?



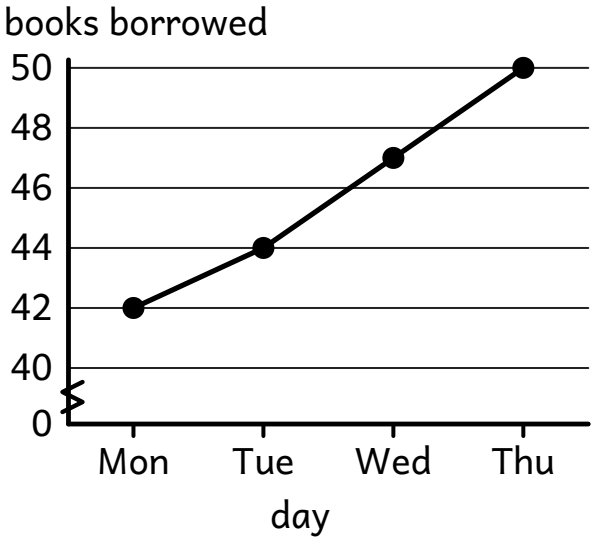
A No. Thursday has only 8 more books than Monday.

B Yes. The bars show 5 times as many books.

C No. Thursday has fewer books than Monday.

Answer at the back of the book.

5. Which sign warns you that the scale of a graph does not start at zero?



A Gridlines across the graph

B A zigzag on the axis

C Points joined by straight lines

Answer at the back of the book.

6. A line graph shows Tuesday at 44 books and Thursday at 50 books. How many more books were borrowed on Thursday?

   Answer at the back of the book.

7. A graph has a scale from 40 to 50, going up in steps of 2. How many steps are there?

   Answer at the back of the book.

8. **From an earlier chapter**

   Review: both groups have a mean of 6. Group X is 5, 6 and 7. Group Y is 2, 6 and 10. How much bigger is the range of Y than the range of X?

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

A line graph shows change over time. Join the points with straight lines. Read the scale before you read the picture. A bar chart starts at 0. A cut line graph needs a zigzag.

Draw: Draw the numbers 42, 44, 47 and 50 as a line graph twice. Start one scale at 0 and the other at 40.

M3.7.6 CHAPTER 6

The language of chance

> **A bag holds 1 black bead and 9 white beads. How likely is it that you pick black?**

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Certain
- ☐ **B** An even chance
- ☐ **C** Unlikely

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Fractions, decimals and percentages, M3.3.7. A probability is a fraction. You change it into a decimal by dividing the top by the bottom.

BY THE END YOU CAN

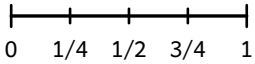
- Place events on a scale from impossible to certain
- Write a probability as a fraction or decimal from 0 to 1

An **event** is something that may happen, such as "a coin lands on heads". Some events cannot happen. Some must happen. Most are in between.

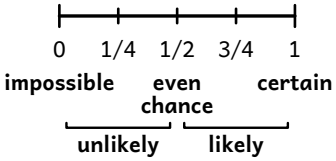
The **probability** of an event is a number from 0 to 1 that says how likely it is. We put these numbers on a line, with the five words in their places.

DRAW IT FIRST

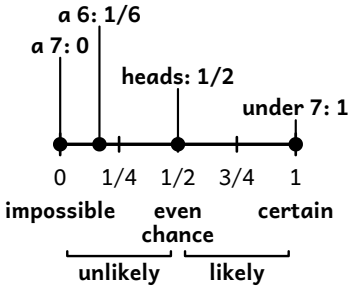
Build the probability line



1 Draw a line. Write 0 at the left end and 1 at the right end. Mark the quarters on it.

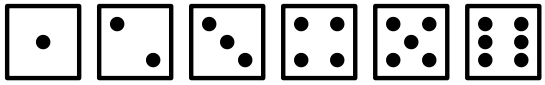


2 Write the words. Impossible is at 0. Even chance is half way. Certain is at 1. Unlikely is between 0 and the middle. Likely is between the middle and 1.



3 Put each event where its probability says. Rolling a 7 on a dice is at 0, and a coin showing heads is half way.

Here are the meanings. **Impossible** is 0: it can never happen. **Certain** is 1: it always happens. **Even chance** is 1/2, or 0.5: it is as likely to happen as not. A dice is the object with faces 1 to 6.



1 2 3 4 5 6

one dice

A dice has six faces. Each face is equally likely to land on top.

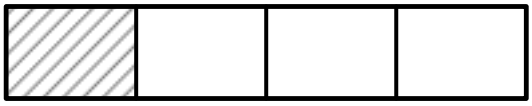
A probability is never below 0 and never above 1. You may write it as a fraction, such as $^1/_4$, or as a decimal, such as 0.25. They are the same number.

WORKED EXAMPLE

A bag holds 4 beads of the same size: 1 black and 3 white. You pick one without looking. Say how likely black is, in words, as a fraction and as a decimal.

1. The beads are the same size, so each bead is equally likely to be picked.
2. One bead out of 4 is black. The chance of black is $^1/_4$.
3. This is less than 0.5, so black is unlikely. As a decimal, 1 ÷ 4 = 0.25.

Answer: Unlikely, $^1/_4$, 0.25



1 black bead in 4

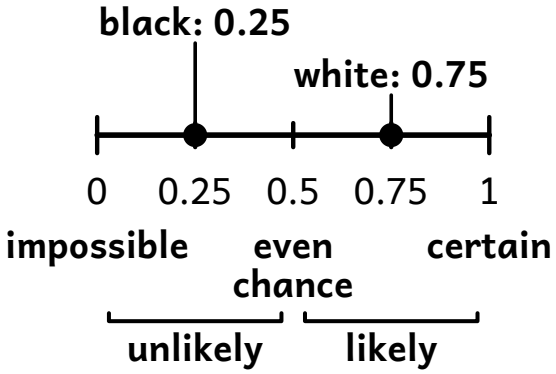
The shaded part is the black bead. Black is 1 part out of 4, which is $^1/_4$.

WORKED EXAMPLE

Use the same bag. How likely is it that you pick a white bead? Say it in words, then as a fraction and a decimal.

1. Three of the 4 beads are white. The chance of white is $^3/_4$.
2. Divide the top by the bottom: 3 ÷ 4 = 0.75.
3. It is more than 0.5, so white is likely. It sits at 0.75 on the line.

Answer: Likely, $^3/_4$, 0.75



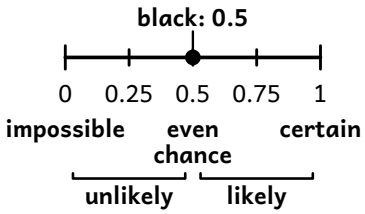
Black is at 0.25, in the unlikely part. White is at 0.75, in the likely part.

SEE IT THREE WAYS

With things

A bag holds 1 black bead and 1 white bead. Shake it and pick one without looking. Each bead is equally likely.

As a picture



In symbols

1 bead out of 2 is black. 1 ÷ 2 = 0.5. This is an even chance.

TRY IT

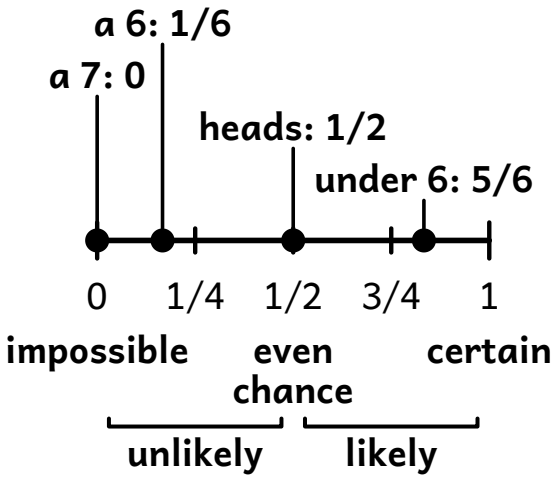
Put events on a line

You need: paper and a pencil, a ruler.

1. Draw a line 20 cm long. Write 0 at the left end and 1 at the right end.
2. Write four events on small pieces of paper. A dice shows 7. A dice shows 6. A coin shows heads. A dice shows a number under 6.
3. Decide where each event goes. Put the paper on the line.
4. Check each one with the words: impossible, unlikely, even chance, likely, certain.

What you will see: A dice showing 7 goes at 0. A 6 is at $^1/_6$, near 0. Heads is at $^1/_2$. A number under 6 is 5 faces out of 6. It is at $^5/_6$, near 1.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~There are two things that can happen, black or white, so each has a chance of $\frac{1}{2}$.~~

Why it feels right: It seems fair to share the chance equally between two answers.

The test that breaks it: A bag holds 1 black bead and 9 white beads. Pick one 100 times, putting it back each time. Black usually comes up about 10 times, far fewer than half.

What is true: Chance is shared equally only when the outcomes are equally likely. Count the beads to find the chance.

KEY WORDS

event
something that may happen.

probability
a number from 0 to 1 that says how likely an event is.

impossible
it can never happen. Its probability is 0.

certain
it always happens. Its probability is 1.

even chance
as likely to happen as not. Its probability is 0.5.

BACK TO YOUR GUESS

The answer is C: Unlikely

Only 1 bead out of 10 is black. Black can happen, but it will not happen most of the time. That is unlikely. In this chapter you will give each of these words a number from 0 to 1.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. You roll a dice. How likely is it that the dice shows 7?

   A Impossible

   B Unlikely

   C Even chance

   Answer at the back of the book.

2. You roll a dice. How likely is it that the dice shows a number less than 7?

   A Likely

   B Certain

   C Even chance

   Answer at the back of the book.

3. Which of these numbers cannot be a probability?

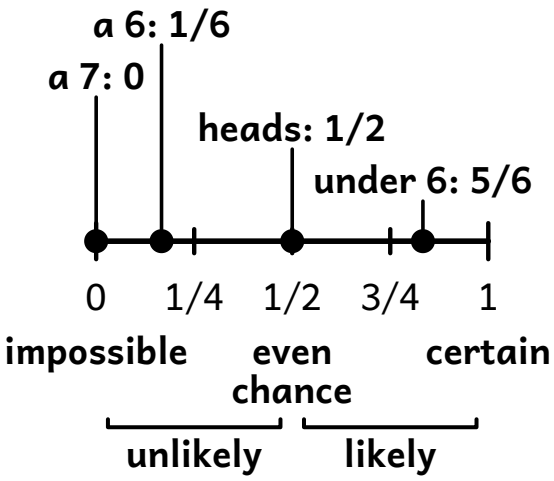
   A 0.3

   B 1

   C 1.2

   Answer at the back of the book.

4. Which event on the line has an even chance?



A heads

B a 6

C under 6

Answer at the back of the book.

5. Write the probability $^1/_4$ as a decimal.

A 1.4

B 0.25

C 0.4

Answer at the back of the book.

6. A bag holds 5 beads of the same size and 2 are red. Write the probability of picking red as a decimal.

Answer at the back of the book.

7. A bag holds 8 beads of the same size and 1 is black. Write the probability of picking black as a decimal.

Answer at the back of the book.

8. **From an earlier chapter**

Review: a line graph shows Monday at 42 books and Wednesday at 47 books. How many more books were borrowed on Wednesday?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Probability is a number from 0 to 1. Impossible is 0. Certain is 1. Even chance is 0.5. Unlikely is below 0.5. Likely is above 0.5. Write it as a fraction or a decimal.

Draw: Draw a line from 0 to 1. Mark 0, 0.5 and 1. Write impossible, even chance and certain under them.

M3.7.7 CHAPTER 7

Working out chances

A dice has six faces. How do I work out the chance of an even number?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** The chance of an even number is $^1/_6$.
- ☐ **B** The chance of an even number is $^1/_3$.
- ☐ **C** The chance of an even number is $^1/_2$.

IF THIS FEELS HARD, READ THIS FIRST

- Level 2, Fractions, M2.4.3. A probability is a fraction: the parts that count out of all the equal parts.

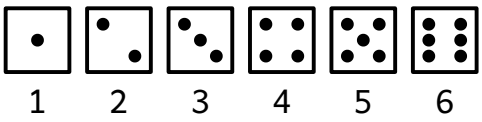
BY THE END YOU CAN

- Find a probability when all outcomes are equally likely
- Find the chance that something does not happen

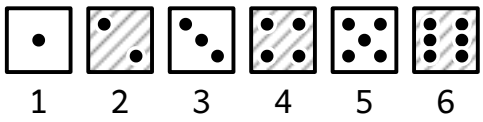
Each possible result is an **outcome**. Rolling a dice has six outcomes: 1, 2, 3, 4, 5 and 6. If a fair dice is rolled, the outcomes are **equally likely**. None of them is favoured.

DRAW IT FIRST

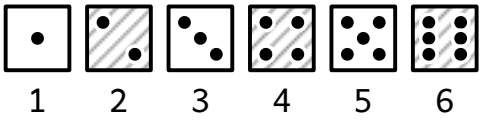
The chance of an even number



1 List all the outcomes of the dice: 1, 2, 3, 4, 5 and 6. There are 6.



2 Mark the outcomes that count. Here they are the even numbers: 2, 4 and 6. There are 3.



chance 3/6 = 1/2

3 Write the chance as a fraction: 3 out of 6 is $^{3}/_{6}$, which is $^{1}/_{2}$ in simplest form.

The rule is: probability = outcomes that count ÷ all outcomes. It works when every outcome is equally likely.

WORKED EXAMPLE

A bag holds 3 red beads and 5 green beads of the same size. You pick one without looking. What is the chance that it is red?

1. There are 3 + 5 = 8 beads in all. Each bead is equally likely to be picked.
2. Three of the beads are red. These are the outcomes that count.
3. The chance of red is 3 out of 8, which is $^{3}/_{8}$.

Answer: $^{3}/_{8}$



3 red beads in 8

Each part is one bead. The shaded parts count. The chance of red is the shaded parts out of all the parts.

Now ask the opposite: what is the chance that something does not happen? Every outcome either counts or does not count. So the two chances add up to 1. The chance of "not" is 1 take away the chance of the event.

WORKED EXAMPLE

Use the same bag of 3 red and 5 green beads. What is the chance that the bead is not red?

1. The chance of red is $^{3}/_{8}$.
2. The beads that are not red are $8 - 3 = 5$.
3. The chance of not red is $^{5}/_{8}$. The two chances add up to 1: $^{3}/_{8} + {}^{5}/_{8} = 1$.

Answer: $^{5}/_{8}$



5 beads that are not red

The shaded parts are now the 5 beads that are not red: 5 parts out of 8.

Some events have more than one stage, such as tossing two coins. List every outcome in a grid. Write one coin down the side and the other across the top. Each cell is one outcome.

	second coin	
first	**H**	**T**
H	HH	HT
T	TH	TT

Two coins give 4 outcomes. HT and TH are different: the first coin is heads in HT and tails in TH.

WORKED EXAMPLE

Two coins are tossed. What is the chance of at least one head?

1. List the outcomes: HH, HT, TH and TT.
2. There are 2 × 2 = 4 outcomes in all, and each is equally likely.
3. The outcomes with at least one head are HH, HT and TH. That is 3 outcomes.
4. The chance is $^3/_4$, which is 3 ÷ 4 = 0.75.

Answer: $^3/_4$

	second coin	
first	**H**	**T**
H	HH	HT
T	TH	TT

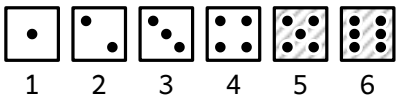
Three cells are hatched out of four. The one plain cell, TT, is the chance of no head: $^1/_4$.

SEE IT THREE WAYS

With things

Roll a dice. List what can happen. Count the faces that show 5 or 6.

As a picture



In symbols

2 faces out of 6 is $^{2}/_{6}$. In simplest form, $^{2}/_{6} = {}^{1}/_{3}$.

TRY IT

List every outcome

You need: paper and a pencil, a coin, if you have one.

1. Write H for heads and T for tails.
2. Write every pair for two coins by putting H and then T after each. You get HH, HT, TH and TT.
3. Count the outcomes. There are 4.
4. Count the outcomes with exactly one head. There are 2.

What you will see: Two of the four outcomes have exactly one head, HT and TH. The chance of exactly one head is 2 out of 4, which is $^{1}/_{2}$.

If you cannot try it, this is what happens.

	second coin	
first	**H**	**T**
H	HH	HT
T	TH	TT

A WRONG IDEA MANY PEOPLE HAVE

~~Two coins have three outcomes: two heads, two tails, or one of each. So each has a chance of $1/3$.~~

Why it feels right: Three outcomes sound like a fair list, and "one of each" is one thing to say.

The test that breaks it: Toss two coins 100 times and tally. "One of each" usually comes up about 50 times, as often as the other two put together.

What is true: "One of each" is two outcomes, HT and TH. The four outcomes HH, HT, TH and TT are equally likely, so each has a chance of $1/4$.

KEY WORDS

outcome
one possible result, such as a 4 on a dice.

equally likely
every outcome has the same chance.

BACK TO YOUR GUESS

The answer is C: The chance of an even number is $1/2$.

A dice has 6 faces, and 3 of them are even: 2, 4 and 6. The chance is 3 out of 6, which is $3/6$. In simplest form that is $1/2$.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. You roll a dice. What is the chance of a 5?

   A $^1/_6$

   B $^1/_5$

   C $^5/_6$

   Answer at the back of the book.

2. A bag holds 2 red, 6 blue and 2 yellow beads of the same size. What is the chance of picking blue?

   A $^6/_4$

   B $^3/_5$

   C $^1/_3$

   Answer at the back of the book.

3. You roll a dice. What is the chance of not rolling a 6?

   A $^1/_6$

   B $^6/_5$

   C $^5/_6$

   Answer at the back of the book.

4. Two coins are tossed. Look at the grid. What is the chance of two tails?

	second coin	
first	**H**	**T**
H	HH	HT
T	TH	TT

   A $^1/_4$

   B $^1/_3$

   C $^1/_2$

   Answer at the back of the book.

5. A spinner has 5 equal sections numbered 1 to 5. What is the chance of a number greater than 3?

   A $^3/_5$

   B $^2/_5$

   C $^2/_3$

   Answer at the back of the book.

6. A bag holds 4 red beads and 8 green beads of the same size. What is the chance of picking red? Write it in simplest form.

   Answer at the back of the book.

7. A bag holds 5 beads of the same size and 1 is black. What is the chance of not picking black? Write it as a decimal.

   Answer at the back of the book.

8. **From an earlier chapter**

   Review: write the probability $^3/_5$ as a decimal.

   Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Probability = outcomes that count ÷ all outcomes, when every outcome is equally likely. The chance of "not" is 1 take away the chance. Two coins have 4 outcomes: HH, HT, TH, TT.

Draw: Draw the 4 outcomes of two coins in a grid. Hatch the cells with exactly one head.

M3.7.8 CHAPTER 8

Experiments and fair games

I tossed a coin 50 times. Why did I not get exactly 25 heads?

YOUR GUESS

Pick one before you read on. It is fine to be wrong.

- ☐ **A** Amina tossed a coin 50 times and got 28 heads. This is normal, because results of an experiment vary a little.
- ☐ **B** Amina tossed a coin 50 times and got 28 heads. The coin must be unfair.
- ☐ **C** Amina tossed a coin 50 times and got 28 heads. She must have counted wrongly.

IF THIS FEELS HARD, READ THIS FIRST

- Level 3, Fractions, decimals and percentages, M3.3.7. Results are compared as fractions and decimals of all the tosses.

BY THE END YOU CAN

- Toss a coin 50 times and compare with what you expected
- Decide if a dice game is fair

Probability says what to **expect**. An **experiment** shows what really happens. A fair coin has a chance of $^1/_2$ of landing on heads. So in 50 tosses we expect about 25 heads.

We say "about". Chance does not count tosses for us. A real run of 50 tosses can give 24, 26 or 28 heads. It gives exactly 25 only about 1 time in 9. Most runs land a few heads away from 25.

DRAW IT FIRST

Toss *a* coin 50 times

H T T H H T H H T H
T H T T H H T H T T
H H T H T H H T T T
T H H T H T H H T H
H T H T H T T H T H

1 Toss *a* coin 50 times. Write H or T each time, in rows of ten.

heads 𝍸 𝍸 𝍸 𝍸 𝍸 |

tails 𝍸 𝍸 𝍸 𝍸 ||||

2 Tally the heads and the tails. Count each tally.

heads 𝍸 𝍸 𝍸 𝍸 𝍸 |

tails 𝍸 𝍸 𝍸 𝍸 ||||

expected 25, found 26

3 Compare with what you expected. Expect 25 heads, because 50 × $^{1}/_{2}$ = 25.

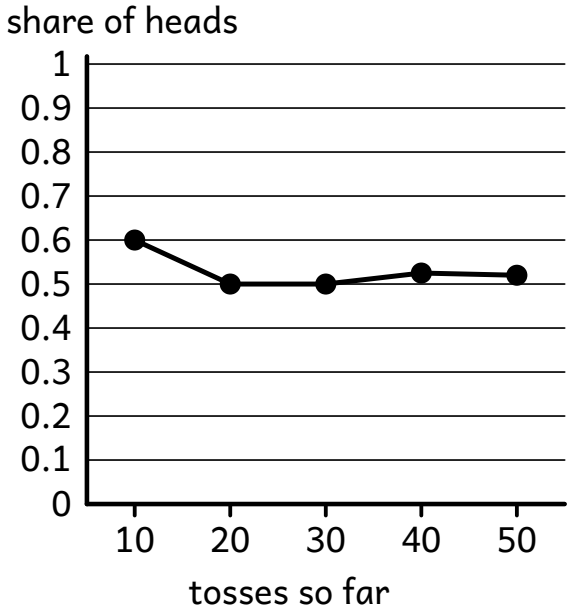
WORKED EXAMPLE

A fair coin is tossed 50 times and gives 26 heads. How many heads would you expect? How far is the result from that?

1. A fair coin has a chance of $^1/_2$ of heads on each toss.
2. Expect about $50 \times {}^1/_2 = 25$ heads.
3. The tosses gave 26 heads. That is $26 - 25 = 1$ more than expected.

Answer: 25 heads expected. The result was 1 more.

Keep tossing and the share of heads usually settles closer to one half. Here is the share of heads after every ten tosses of the same 50.



The share of heads starts at 0.6 and then stays close to 0.5.

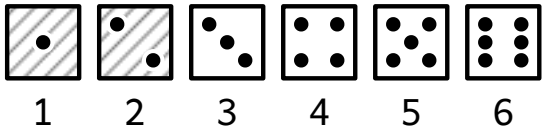
Now think about games. A game is **fair** when every player has the same chance of winning. To check, count the outcomes that win for each player. Do not guess from how the rules sound.

WORKED EXAMPLE

Ravi and Leila roll a dice. Ravi wins on 1 or 2. Leila wins on 3, 4, 5 or 6. Is the game fair? If not, how can you make it fair?

1. A dice has 6 equally likely faces. Ravi wins on 1 and 2. That is 2 faces.
2. Leila wins on 3, 4, 5 and 6. That is 4 faces.
3. The chances are $^2/_6$ and $^4/_6$. They are not equal, so the game is not fair.
4. To make it fair, give each player the same number of faces: 6 ÷ 2 = 3.

Answer: No. Ravi has 2 faces and Leila has 4. Each needs 3 faces.



Ravi has 2 hatched faces. Leila has 4 plain faces. Leila is twice as likely to win.

WORKED EXAMPLE

Amina and Kofi roll two dice and add the numbers. Amina wins if the sum is 7. Kofi wins if the sum is 9 or more. Is the game fair?

1. Two dice have 6 × 6 = 36 equally likely outcomes. The grid shows the sum of each.
2. Amina wins on a sum of 7. That is 6 outcomes.
3. Kofi wins on a sum of 9, 10, 11 or 12: 4 + 3 + 2 + 1 = 10 outcomes.
4. Amina has 6 chances in 36 and Kofi has 10. They are not equal, so the game is not fair.

Answer: No. Kofi has 10 chances in 36 and Amina has 6.

	second dice					
first	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Amina's sums. Six cells out of 36 are hatched, so her chance is $^{6}/_{36}$, which is $^{1}/_{6}$.

	second dice					
first	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Kofi's sums. Ten cells out of 36 are hatched, so his chance is $^{10}/_{36}$, which is $^{5}/_{18}$.

SEE IT THREE WAYS

With things

Toss a coin 10 times and count the heads. You may get 6. You expected 5.

As a picture

Ⓗ Ⓣ Ⓣ Ⓗ Ⓗ Ⓣ Ⓗ Ⓗ Ⓣ Ⓗ

In symbols

Expected: $10 \times {}^{1}/_{2} = 5$ heads. Found: 6 heads. The difference is $6 - 5 = 1$.

TRY IT

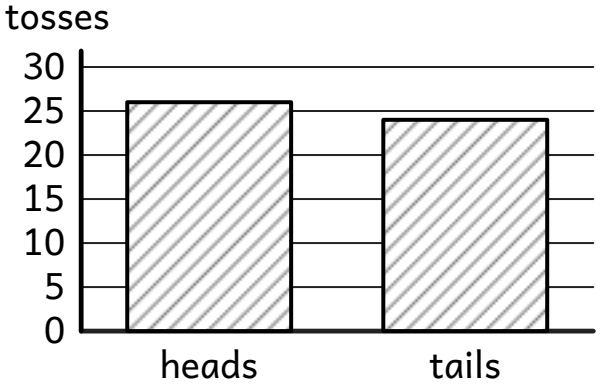
Your own 50 tosses

You need: a coin, paper and a pencil.

1. Write "heads" and "tails" on your paper. Toss the coin 50 times.
2. Make a tally mark for each result. Every fifth mark crosses the four before it.
3. Count the heads and the tails. Add them. They must make 50.
4. Compare the heads with 25. Is the difference small?

What you will see: A typical run gives about 25 heads and 25 tails, plus or minus a few. In the run shown, heads has 26 and tails has 24. Your numbers will differ. That is how chance works.

If you cannot try it, this is what happens.



A WRONG IDEA MANY PEOPLE HAVE

~~If you toss a fair coin 50 times, you must get exactly 25 heads.~~

Why it feels right: A chance of one half sounds like a promise of half.

The test that breaks it: Toss a coin 10 times and count the heads. Do it again. You will not always get 5.

What is true: A probability says what is expected on average. Real results vary a little. With more tosses the share of heads usually gets closer to one half.

KEY WORDS

experiment
a test you really do and record, such as 50 tosses.

expect
what probability says should happen on average.

fair
a game where every player has the same chance of winning.

BACK TO YOUR GUESS

The answer is A: Amina tossed a coin 50 times and got 28 heads. This is normal, because results of an experiment vary a little.

A fair coin lands on heads about half the time. That is what we expect. Chance does not follow a plan. Real results are usually close to 25 heads. Only about 1 run in 9 gives exactly 25.

CHECK YOURSELF

Try each one before you look. A wrong answer tells you where to go next.

1. A coin is tossed 50 times and gives 28 heads. What does this show?

   A The coin must be unfair.

   B The tosses must have been counted wrongly.

   C This is a normal result for a fair coin.

   Answer at the back of the book.

2. A fair coin is tossed 100 times. About how many heads do you expect?

   A About 25 heads

   B About 50 heads

   C About 100 heads

   Answer at the back of the book.

3. Two friends roll a dice. Which game is fair?

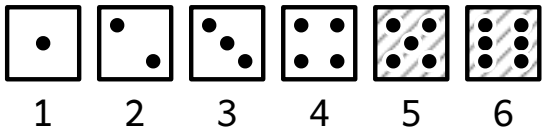
   A Ana wins on an even number. Ravi wins on an odd number.

   B Ana wins on 1 or 2. Ravi wins on 3, 4, 5 or 6.

   C Ana wins on a 6. Ravi wins on any other number.

   Answer at the back of the book.

4. Mei rolls a dice. She wins on a 5 or a 6. The hatched faces are the faces she wins on. What is the chance that Mei wins?



   A $^1/_2$

   B $^2/_5$

   C $^1/_3$

   Answer at the back of the book.

5. Sipho tosses a coin 10 times and gets 7 heads. Leila tosses a coin 100 times and gets 52 heads. Whose result is a better guide to the true chance of heads?

   A Sipho's, because 7 out of 10 is a high share of heads.

   B Leila's, because more tosses give a better guide.

   C Both are equally good.

   Answer at the back of the book.

6. A fair coin is tossed 80 times. About how many heads do you expect?

   Answer at the back of the book.

7. A coin is tossed 50 times and lands on heads 26 times. What share of the tosses were heads? Write it as a decimal.

   Answer at the back of the book.

8. **From an earlier chapter**

Review: a bag holds 7 beads of the same size and 2 are red. What is the chance of not picking red?

Answer at the back of the book.

KEEP IT

Copy this into your exercise book.

Expect about half heads from a fair coin, but results vary a little. More tosses usually come closer. A game is fair when each player has the same chance. Count the winning outcomes.

Draw: Draw the grid of two dice sums with the sums of 7 hatched. Write 6 out of 36.

Key words, A to Z

Every key word in this book, what it means, and the chapter that teaches it.

average
one number that stands for a whole list. M3.7.3

bar chart
a chart with one bar for each class, as high as its count. M3.7.2

certain
it always happens. Its probability is 1. M3.7.6

class
one group of values, such as 10 to 19. M3.7.1

counted data
data found by counting, always a whole number. M3.7.1

data
the facts and numbers you collect. M3.7.1

equally likely
every outcome has the same chance. M3.7.7

even chance
as likely to happen as not. Its probability is 0.5. M3.7.6

event
something that may happen. M3.7.6

expect
what probability says should happen on average. M3.7.8

experiment
a test you really do and record, such as 50 tosses. M3.7.8

fair
a game where every player has the same chance of winning. M3.7.8

impossible
it can never happen. Its probability is 0. M3.7.6

line graph
a graph with points joined by lines, for data that changes over time. M3.7.5

mean
the total shared equally: add them up, then divide by how many. M3.7.3

measured data
data found with a ruler, scale or clock. M3.7.1

median
the middle number when the list is in order. M3.7.3

misleading
giving a wrong picture, even if the numbers are right. M3.7.5

mode
the number that appears most often. A list can have no mode, or two. M3.7.3

outcome
one possible result, such as a 4 on a dice. M3.7.7

outlier
a value far from the rest. M3.7.4

pie chart
a circle cut into sectors, one for each part of a whole. M3.7.2

probability
a number from 0 to 1 that says how likely an event is. M3.7.6

range
the largest number take away the smallest. M3.7.3

scale
the numbers along an axis, with equal steps. M3.7.5

sector
one slice of a circle, drawn with an angle. M3.7.2

spread
how far apart the values are, shown by the range. M3.7.4

survey
asking many people the same question. M3.7.1

Answers

M3.7.1. Collecting data

1. **How do you usually get to school: walk, bicycle, bus, car or other?**

   It asks about one thing, it gives choices, and every answer can be counted.

   - **If you chose B:** That asks about feelings, not about travel. A question must ask about the thing you want to know. Go to M3.7.1.
   - **If you chose C:** This is too open. One child says "fast" and another says "in the morning". Those answers cannot be counted. Go to M3.7.1.

2. **0 to 9, 10 to 19, 20 to 29**

   Each class holds 10 whole numbers. One class ends at 9 and the next starts at 10. There is no gap and no overlap.

   - **If you chose A:** The answer 10 fits both 0 to 10 and 10 to 20. Classes must not overlap. Go to M3.7.1.
   - **If you chose B:** The answer 10 fits no class here. Classes must leave no gap. Go to M3.7.1.

3. **The mass of a bag of rice**

   You weigh the rice with a scale. A mass can be any size, such as 2.5 kg. The other two are counted.

   - **If you chose A:** You count pets: 0, 1, 2. Counted data is a whole number. Go to M3.7.1.
   - **If you chose C:** You count players one by one. That is counted data. Go to M3.7.1.

4. **10 to 19**

   19 is at least 10 and at most 19, so it fits the class 10 to 19.

   - **If you chose B:** The class 0 to 9 stops at 9. The answer 19 is bigger than 9. Go to M3.7.1.
   - **If you chose C:** The class 20 to 29 starts at 20. The answer 19 is smaller than 20. Go to M3.7.1.

5. **7 children**

   The number under the box named 10 to 19 is 7.

   - **If you chose A:** 19 is the end of the class, not a count. Read the number under the box. Go to M3.7.1.
   - **If you chose B:** That is the count for the class 0 to 9. Look at the box named 10 to 19. Go to M3.7.1.

6. **3 children**

   The answers 12, 14 and 17 fit 10 to 19. That is 3 children.

7. **25 children**

   Add the counts: 6 + 11 + 8 = 25.

M3.7.2. Bar charts and pie charts

1. **90°**

   One child is 360 ÷ 24 = 15°. Six children is 6 × 15 = 90°.

   - **If you chose B:** 15° is the angle for one child. The rice sector holds 6 children, so multiply by 6. Go to M3.7.2.
   - **If you chose C:** That is the count, not the angle. Work out one child first: 360 ÷ 24 = 15°. Go to M3.7.2.

2. **0 to 9**

   The tallest bar is the first one, and it stands over the class 0 to 9.

   - **If you chose A:** The class 20 to 29 has the biggest numbers of books, but the fewest children. The class with the most children has the tallest bar. Go to M3.7.2.
   - **If you chose B:** 9 is the height of the tallest bar. The question asks for the name of the class under it. Go to M3.7.2.

3. **A chart of the number of books each child read, grouped into classes**

   Counted data jumps from one whole number to the next, so its bars have a gap. Measured data has no jump, so its bars touch.

   - **If you chose A:** Heights are measured, so they run on with no jump from one class to the next. The bars touch. Go to M3.7.2.
   - **If you chose C:** Books read is counted data. It jumps from 9 to 10 with nothing between, so its bars stand apart. Go to M3.7.1.

4. **$^1/_3$**

   120 out of 360 is $^1/_3$, because 360 ÷ 120 = 3.

   - **If you chose B:** Half of the circle is 180°. The walk sector is smaller than that. Go to M3.7.2.
   - **If you chose C:** 12 is the number of children who walk. The fraction uses the angle: 120 out of 360. Go to M3.7.2.

5. **Together the sectors make one whole turn of the circle.**

   A pie chart shows one whole shared out. A whole is one full turn, which is 360°.

   - **If you chose A:** Sectors can be any size. Only the total is fixed. It is one whole turn. Go to M3.7.2.
   - **If you chose B:** The number of children changes from survey to survey. The angles are scaled so they always fill the circle. Go to M3.7.2.

6. **6 children**

   One child is 360 ÷ 36 = 10°. The bus sector is 60°, and 60 ÷ 10 = 6 children.

   - **If you chose A:** That is the angle, and it is more than the 36 children in all. Divide the angle by 10, the angle of one child. Go to M3.7.2.
   - **If you chose C:** 10° is the angle of one child. Divide 60° by it to count the children. Go to M3.7.2.

7. **60°**

   One child is 360 ÷ 30 = 12°. Five children is 5 × 12 = 60°.

8. **$^1/_8$**

   45 out of 360 is $^1/_8$, because 360 ÷ 45 = 8.

9. **10 to 19**

   14 is at least 10 and at most 19, so it fits the class 10 to 19. Classes have no gap and no overlap.

M3.7.3. Mean, median and mode

1. **5**

   2 + 3 + 10 = 15. Shared between 3 numbers, 15 ÷ 3 = 5.

   - **If you chose A:** That is the middle number, the median. The mean is the total shared equally: add them, then divide. Go to M3.7.3.
   - **If you chose B:** That is the total. You must still share it between the 3 numbers. Go to M3.7.3.

2. **6**

   In order: 2, 4, 6, 8, 9. The middle number is 6.

   - **If you chose A:** That is the middle number as written, but the list is not in order. Order it first. Go to M3.7.3.
   - **If you chose C:** That is the mean. The median is the middle of the ordered list. Go to M3.7.3.

3. **3 goals**

   The tallest stack has 3 dots, at 3 goals. So the mode is 3.

   - **If you chose B:** That is the biggest number, not the most common. The mode is the tallest stack of dots. Go to M3.7.3.
   - **If you chose C:** There are 2 dots at 5, but 3 dots at another number. The mode has the most dots. Go to M3.7.3.

4. **15**

   The largest is 20 and the smallest is 5. 20 − 5 = 15.

   - **If you chose A:** That is the largest number. The range is the largest take away the smallest. Go to M3.7.3.
   - **If you chose B:** You added the largest and the smallest. The range is a difference, so take away. Go to M3.7.3.

5. **9**

   The total is 6 × 3 = 18. The two known numbers make 4 + 5 = 9. The third number is 18 − 9 = 9.

   - **If you chose A:** That is the mean, not the missing number. The total of the 3 numbers is 6 × 3 = 18. Go to M3.7.3.
   - **If you chose C:** You added 4, 5 and 6 as if the mean were a third number. Find the total, 18, then take away 4 and 5. Go to M3.7.3.

6. **5**

   3 + 7 + 4 + 6 = 20. Shared between 4 numbers, 20 ÷ 4 = 5.

7. **7**

   In order: 3, 5, 7, 9, 12. The middle number is 7.

8. **72°**

   One child is 360 ÷ 40 = 9°. Eight children is 8 × 9 = 72°.

M3.7.4. Comparing two groups

1. **Group B is more spread out.**

   A bigger range means the values are more spread out. B has the bigger range.

   - **If you chose A:** A small range means the values are close together. Group A has the smaller range, so it is the steadier group. Go to M3.7.4.
   - **If you chose B:** The same mean does not make the same group. The ranges differ, so the groups differ. Go to M3.7.4.

2. **The median, 6 minutes**

   The median is the middle value, 6. The outlier 46 does not move it. The mean is 84 ÷ 7 = 12.

   - **If you chose A:** The mean of 12 is bigger than six of the seven times. The 46 minutes is an outlier and pulls the mean up. Go to M3.7.4.
   - **If you chose C:** The range is not an average. It tells you the spread, not what is typical. Go to M3.7.3.

3. **Group Q, range 8**

   Group Q has dots from 1 to 9, so its range is 9 − 1 = 8. Group P has dots from 4 to 6, so its range is 2.

   - **If you chose B:** Group P has all its dots between 4 and 6. It is the steadier group. Look at the group with the dots furthest apart. Go to M3.7.4.
   - **If you chose C:** 9 is the largest value, not the range. The range is the largest take away the smallest: 9 − 1. Go to M3.7.3.

4. **The mode**

   The mode is the value that appears most often. That is the size most children wear.

   - **If you chose A:** The mean could be a size like 5.3, which no shoe has. It also does not tell you which size sold most. Go to M3.7.4.
   - **If you chose B:** The median is the middle size. The most popular size can be somewhere else. Go to M3.7.4.

5. **Team X**

   Both teams score about the same on average. Team X has the smaller range, so its scores stay closer together.

   - **If you chose A:** Team Y has the bigger range, so its scores jump about more. Steady means a small range. Go to M3.7.4.
   - **If you chose C:** The means are equal, but the ranges are not. The range is the number that shows steadiness. Go to M3.7.4.

6. **8**

   Group B is more spread out. Its range is 9 − 1 = 8. Group A has a range of 6 − 4 = 2.

7. **2 goals**

   Take the means away: 7 − 5 = 2.

8. **7**

   In order: 4, 6, 8, 12. The two middle numbers are 6 and 8. Halfway between them is (6 + 8) ÷ 2 = 7.

M3.7.5. Line graphs and honest charts

1. **It makes small differences between the bars look big.**

   The bottom of the scale is cut away. The bars that remain look very different in height, even if the numbers are close.

   - **If you chose B:** The numbers do not change. The bars are only cut at the bottom, so they look different. Go to M3.7.5.
   - **If you chose C:** It does the opposite. When the bottom is cut away, the gaps between the bars look larger. Go to M3.7.5.

2. **47 books**

   The Wednesday point is halfway between the line for 46 and the line for 48. Halfway is 47.

   - **If you chose A:** 46 is the line just below the point. The point is halfway up to the next line, so read between the lines. Go to M2.7.6.
   - **If you chose C:** 48 is the line just above the point. The Wednesday point does not reach it. Go to M2.7.6.

3. **A line graph**

   A line graph puts time along the bottom. It joins the points, so you can see the temperature rise and fall.

   - **If you chose A:** Class bars show how many values fall in each group. They do not show change through time. Go to M3.7.1.
   - **If you chose B:** A pie chart shows how one whole is shared at one moment. Temperature changes over time, so use a graph with time along the bottom. Go to M3.7.2.

4. **No. Thursday has only 8 more books than Monday.**

   Monday is 42 and Thursday is 50. The difference is 50 − 42 = 8. The cut scale made it look much bigger.

   - **If you chose B:** The scale starts at 40, so only the top of each bar shows. Read the numbers: Monday is 42 and Thursday is 50. Go to M3.7.5.
   - **If you chose C:** Thursday's bar is the taller one. It has more books, but not 5 times as many. Go to M3.7.5.

5. **A zigzag on the axis**

   A zigzag on the axis means part of the scale is left out. Always check the first number on the axis too.

   - **If you chose A:** Gridlines help you read values. They say nothing about where the scale starts. Go to M3.7.5.
   - **If you chose C:** Joining points is how a line graph is drawn. It does not warn that the scale is cut. Go to M3.7.5.

6. **6 books**

   Take away: 50 − 44 = 6.

7. **5 steps**

   The scale covers 50 − 40 = 10. Each step is 2, so 10 ÷ 2 = 5 steps.

8. **6**

   The range of X is 7 − 5 = 2. The range of Y is 10 − 2 = 8. The difference is 8 − 2 = 6.

M3.7.6. The language of chance

1. **Impossible**

   A dice shows 1, 2, 3, 4, 5 or 6. A 7 can never show, so the probability is 0.

   - **If you chose B:** Unlikely means it can still happen. A dice has the faces 1 to 6 and no face with 7, so it cannot happen. Go to M3.7.6.
   - **If you chose C:** An even chance is a probability of 0.5. A 7 can never show, so its probability is 0. Go to M3.7.6.

2. **Certain**

   All six faces, 1 to 6, are less than 7. It always happens, so the probability is 1.

   - **If you chose A:** Likely means it may fail to happen. Every face of a dice is less than 7, so it always happens. Go to M3.7.6.
   - **If you chose C:** Only half the faces would give an even chance. Here all 6 faces count. Go to M3.7.6.

3. **1.2**

   A probability is never above 1, because 1 means certain. Nothing is more than certain.

   - **If you chose A:** The number 0.3 is between 0 and 1, so it can be a probability. Go to M3.7.6.
   - **If you chose B:** The number 1 is the biggest probability. It means certain. Go to M3.7.6.

4. **heads**

   Heads is half way along the line, at 1/2. That is an even chance.

   - **If you chose B:** A 6 is at 1/6, close to 0. That is unlikely, not an even chance. Go to M3.7.6.
   - **If you chose C:** Under 6 is at 5/6, close to 1. That is likely, not an even chance. Go to M3.7.6.

5. **0.25**

   Divide the top by the bottom: 1 ÷ 4 = 0.25.

   - **If you chose A:** You wrote the 1 and the 4 side by side. The fraction $^1/_4$ means 1 divided by 4. That is less than 1. Go to M3.3.7.
   - **If you chose C:** That is 4 tenths, which is $^2/_5$. For $^1/_4$, work out 1 ÷ 4. Go to M3.3.7.

6. **0.4**

   Red is 2 out of 5 beads, which is $^2/_5$. Divide: 2 ÷ 5 = 0.4.

7. **0.125**

   Black is 1 out of 8 beads. Divide: 1 ÷ 8 = 0.125.

8. **5 books**

   Take away: 47 − 42 = 5.

M3.7.7. Working out chances

1. **$^1/_6$**

   There is 1 face with a 5 and there are 6 faces in all. The chance is $^1/_6$.

   - **If you chose B:** A dice has 6 faces, not 5. There are 6 equally likely outcomes in all. Go to M3.7.7.
   - **If you chose C:** That is the chance of not rolling a 5. One face out of 6 shows a 5. Go to M3.7.7.

2. **$^3/_5$**

   There are 2 + 6 + 2 = 10 beads. Six are blue, so the chance is $^6/_{10}$. In simplest form that is $^3/_5$.

   - **If you chose A:** You compared blue with the other beads. A chance is the beads that count out of all the beads. It cannot be more than 1. Go to M3.7.7.
   - **If you chose C:** There are 3 colours, but the colours are not equally likely. Count the beads: 6 blue out of 10. Go to M3.7.6.

3. **$^{5}/_{6}$**

   The chance of a 6 is $^{1}/_{6}$. The chance of not a 6 is 1 take away $^{1}/_{6}$, which is $^{5}/_{6}$.

   - **If you chose A:** That is the chance of rolling a 6. The question asks for the chance of not rolling a 6. Go to M3.7.7.
   - **If you chose B:** A chance is never more than 1. Five faces count, out of 6 faces in all. Go to M3.7.7.

4. **$^{1}/_{4}$**

   One outcome out of the 4 is TT, so the chance is $^{1}/_{4}$.

   - **If you chose B:** HT and TH are two different outcomes, so there are 4 equally likely outcomes, not 3. Go to M3.7.7.
   - **If you chose C:** Half would be 2 cells out of 4. Only 1 cell, TT, shows two tails. Go to M3.7.7.

5. **$^{2}/_{5}$**

   The numbers greater than 3 are 4 and 5, so 2 sections count. There are 5 sections in all. The chance is $^{2}/_{5}$.

   - **If you chose A:** That counts 1, 2 and 3, the numbers that are 3 or less. "Greater than 3" means 4 and 5. Go to M3.7.7.
   - **If you chose C:** You compared the 2 numbers that win with the 3 that do not. Divide by all 5 sections. Go to M3.7.7.

6. **$^{1}/_{3}$**

   There are 4 + 8 = 12 beads. Four are red, so the chance is $^{4}/_{12}$. In simplest form that is $^{1}/_{3}$.

7. **0.8**

   The chance of black is $^{1}/_{5}$, which is 0.2. The chance of not black is 1 − 0.2 = 0.8.

8. **0.6**

   Divide the top by the bottom: 3 ÷ 5 = 0.6.

M3.7.8. Experiments and fair games

1. **This is a normal result for a fair coin.**

   We expect about 25 heads. 28 is close to 25, so the coin is behaving like a fair coin.

   - **If you chose A:** A fair coin is not stuck on exactly half. 28 is close to the 25 we expect, so a fair coin often gives a result like this. Go to M3.7.8.
   - **If you chose B:** Nothing is wrong. Chance gives results near the expected number, not exactly on it. Go to M3.7.8.

2. **About 50 heads**

   The chance of heads is $^1/_2$. Half of 100 is $100 \times {}^1/_2 = 50$.

   - **If you chose A:** That is a quarter of 100. A fair coin lands on heads half of the time, not a quarter. Go to M3.7.7.
   - **If you chose C:** That would be heads every time. Each toss has a chance of only $^1/_2$ of heads. Go to M3.7.6.

3. **Ana wins on an even number. Ravi wins on an odd number.**

   Even numbers are 2, 4 and 6. Odd numbers are 1, 3 and 5. Each player has 3 faces out of 6, so the chances are equal.

   - **If you chose B:** Ana wins on 2 faces and Ravi wins on 4 faces. Ravi has twice the chance, so it is not fair. Go to M3.7.8.
   - **If you chose C:** Ana has 1 face and Ravi has 5. Ravi is much more likely to win. Go to M3.7.8.

4. **$^1/_3$**

   Two faces out of 6 win. $^2/_6$ in simplest form is $^1/_3$.

   - **If you chose A:** Half would be 3 faces out of 6. Mei wins on only 2 faces. Go to M3.7.7.
   - **If you chose B:** A dice has 6 faces, not 5. Divide the 2 winning faces by all 6 faces. Go to M3.7.7.

5. **Leila's, because more tosses give a better guide.**

   With more tosses the share of heads usually gets closer to one half. Leila's share, 52 out of 100, is very close to it.

   - **If you chose A:** A short run can wander far from the expected result. Ten tosses is too few to trust. Go to M3.7.8.
   - **If you chose C:** More tosses give a more reliable guide. 100 tosses tell you more than 10. Go to M3.7.8.

6. **40 heads**

   The chance of heads is $^1/_2$. Half of 80 is $80 \times {}^1/_2 = 40$.

7. **0.52**

   Divide the heads by the tosses: $26 \div 50 = 0.52$.

8. **$^5/_7$**

   The beads that are not red are $7 - 2 = 5$. The chance is $^5/_7$.

About this book

For learners who can draw a bar chart from a frequency table and who know what a fraction like 3/4 means. In many schools this fits children of about 11 to 13, and older learners who never met statistics can start here too. Each chapter takes one or two short sessions. A learner can work alone: every exercise has its answer and the reason for it. Each chapter also has a short page for the adult, with three questions to ask.

Written and drawn by attnom. Edition 1.0, 2026-11-12.

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